

Sliding DCT

Mathematical Models and Methods for Image Processing

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<https://boracchi.faculty.polimi.it/teaching/MMMIP.htm>

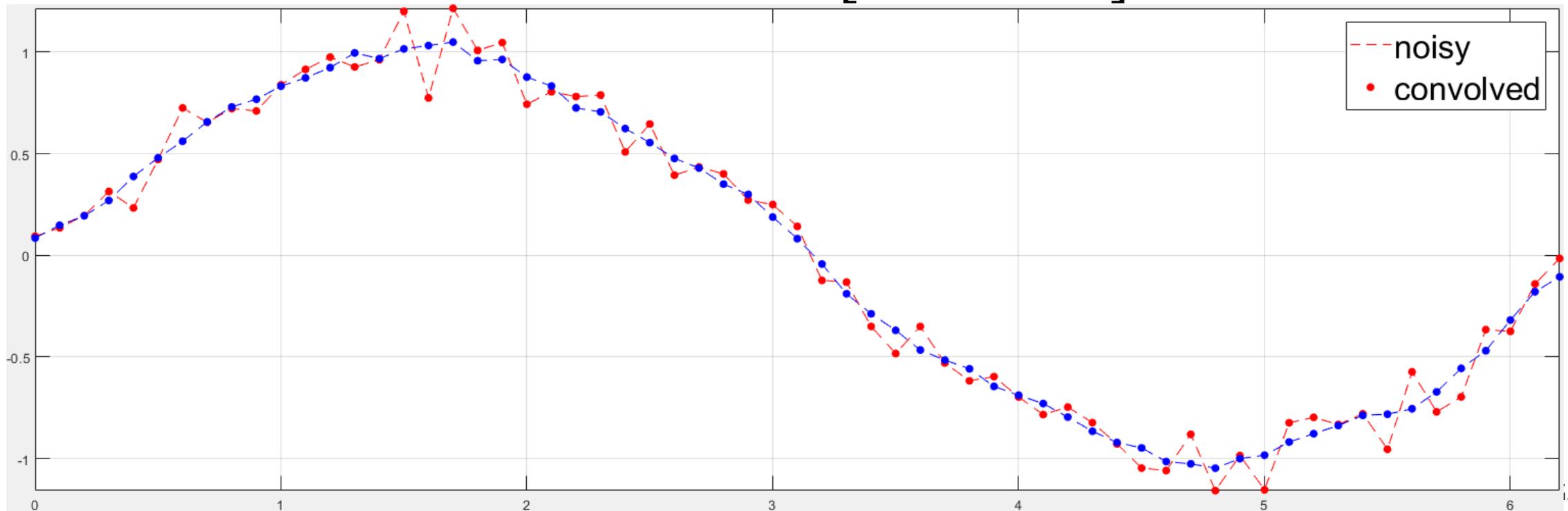
March 3rd 2026

Denoising by Smoothing

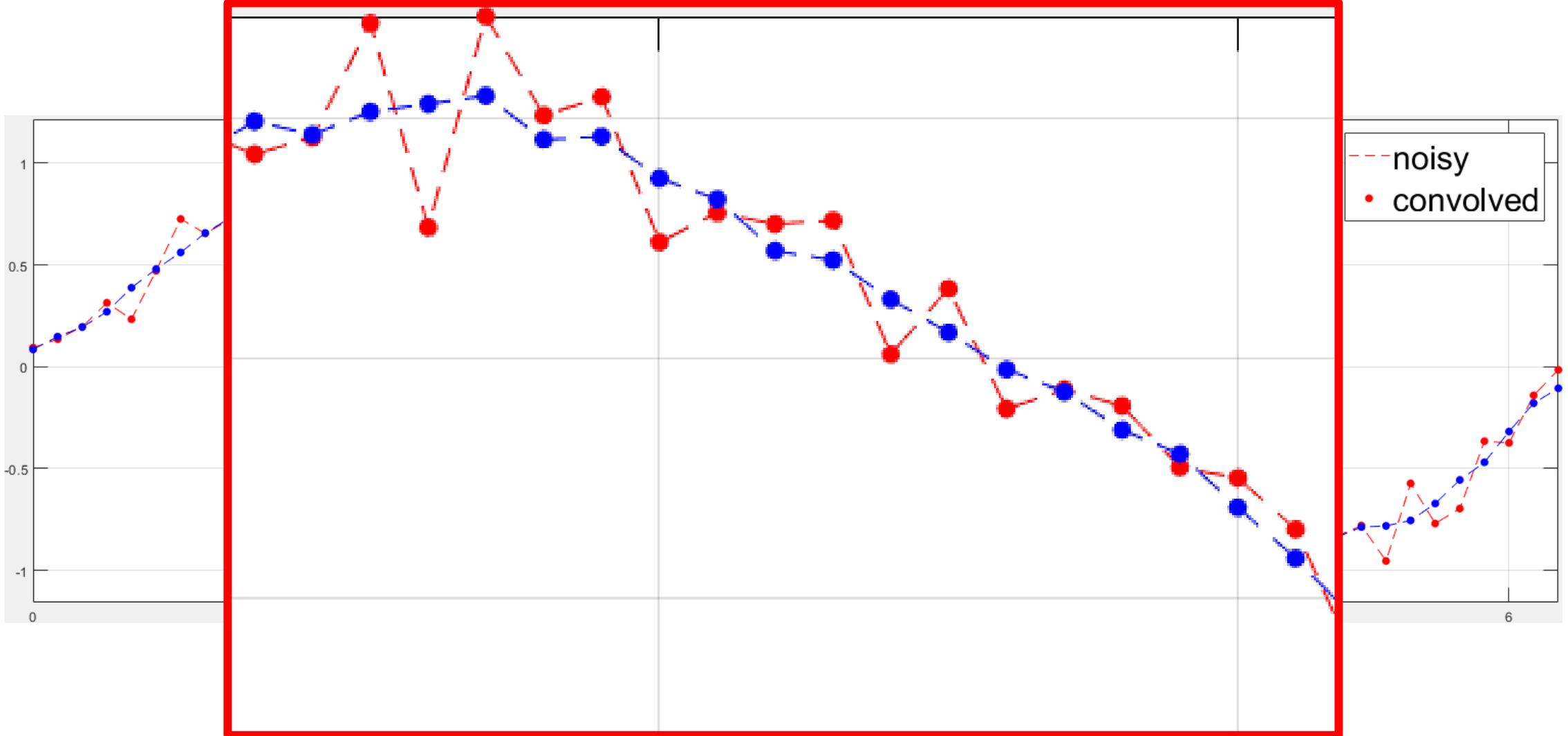
Denoising by Smoothing

$$\hat{y}(x_0) = (z \otimes \mathbf{w})(x_0) = \sum_{m=-M/2}^{M/2} z(x_0 - m) \mathbf{w}(m)$$

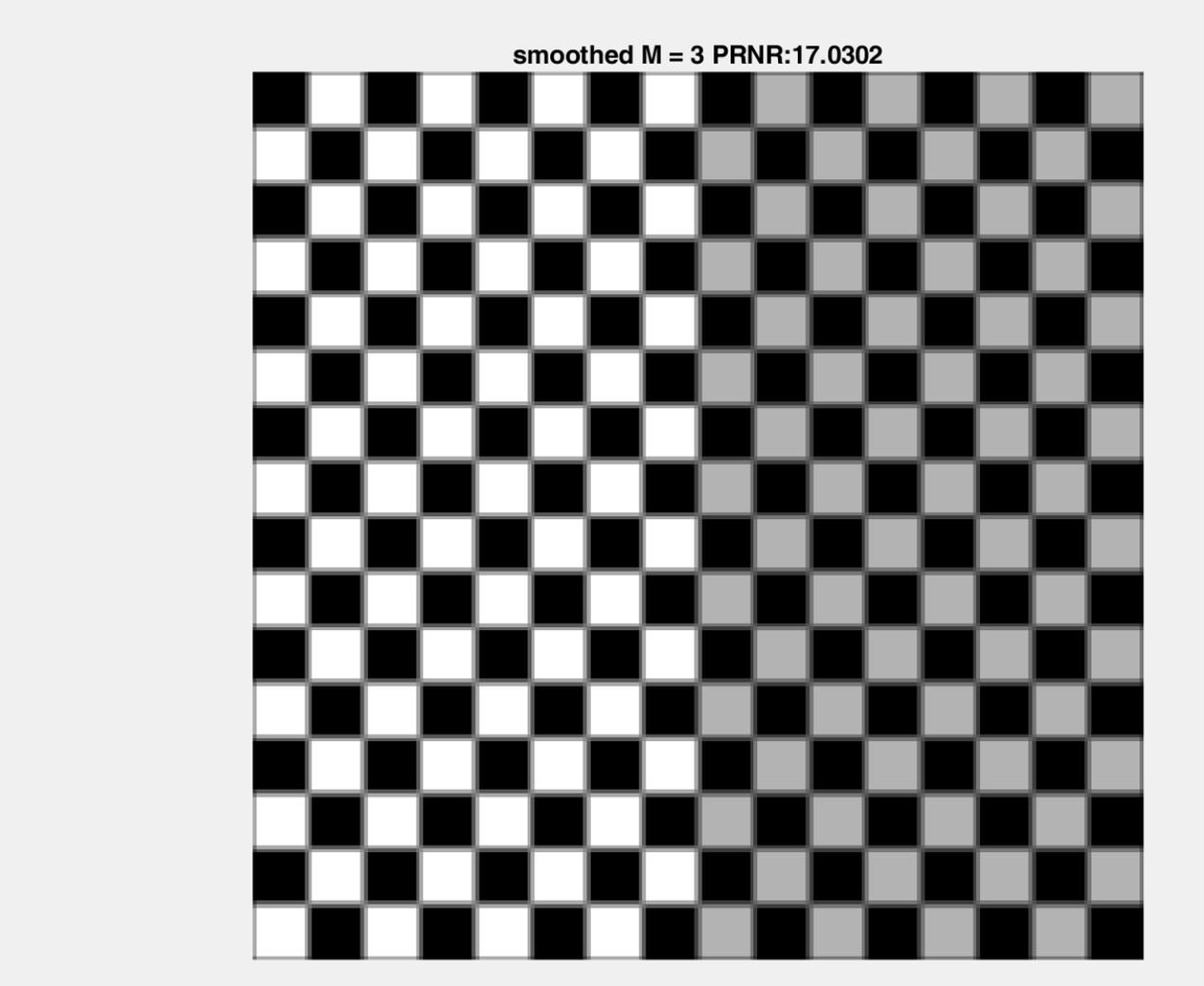
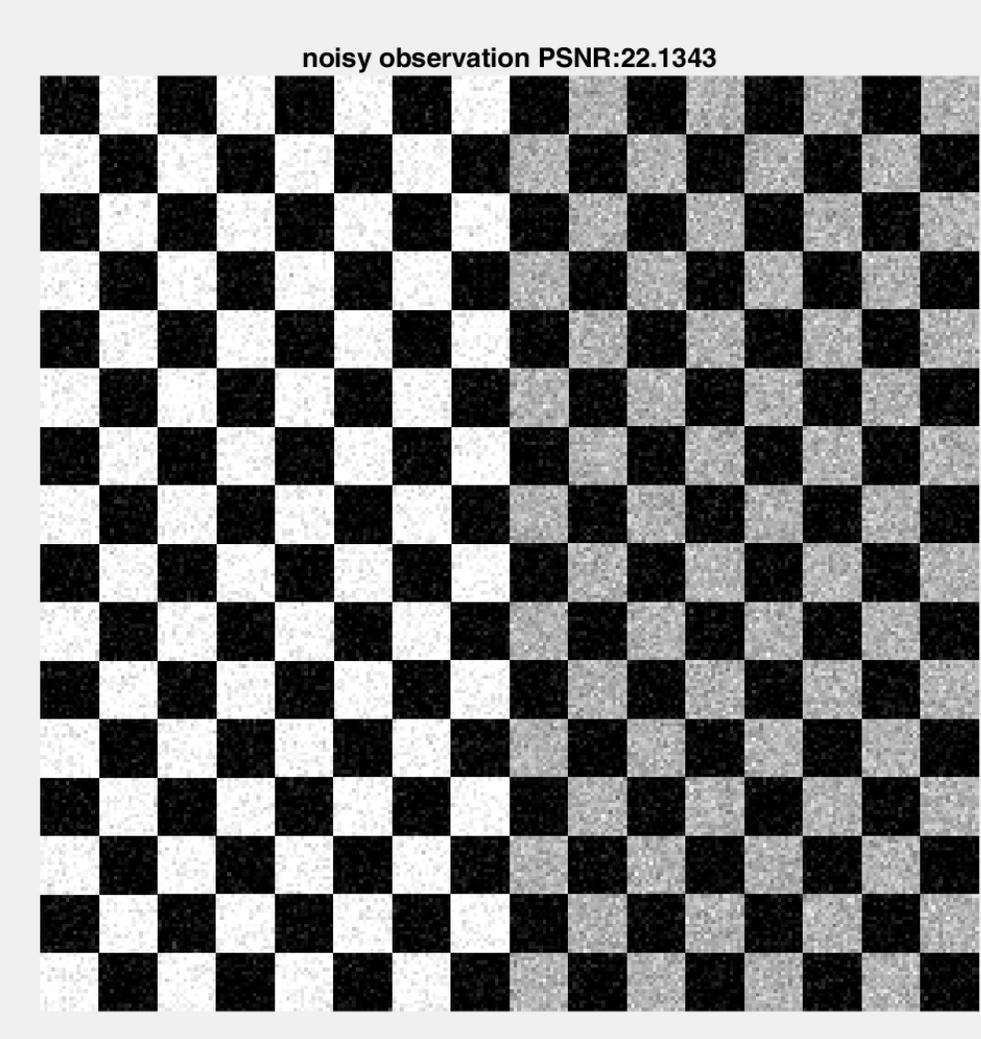
$$y(x) = \sin(x), \mathbf{w} = \left[\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \right]$$



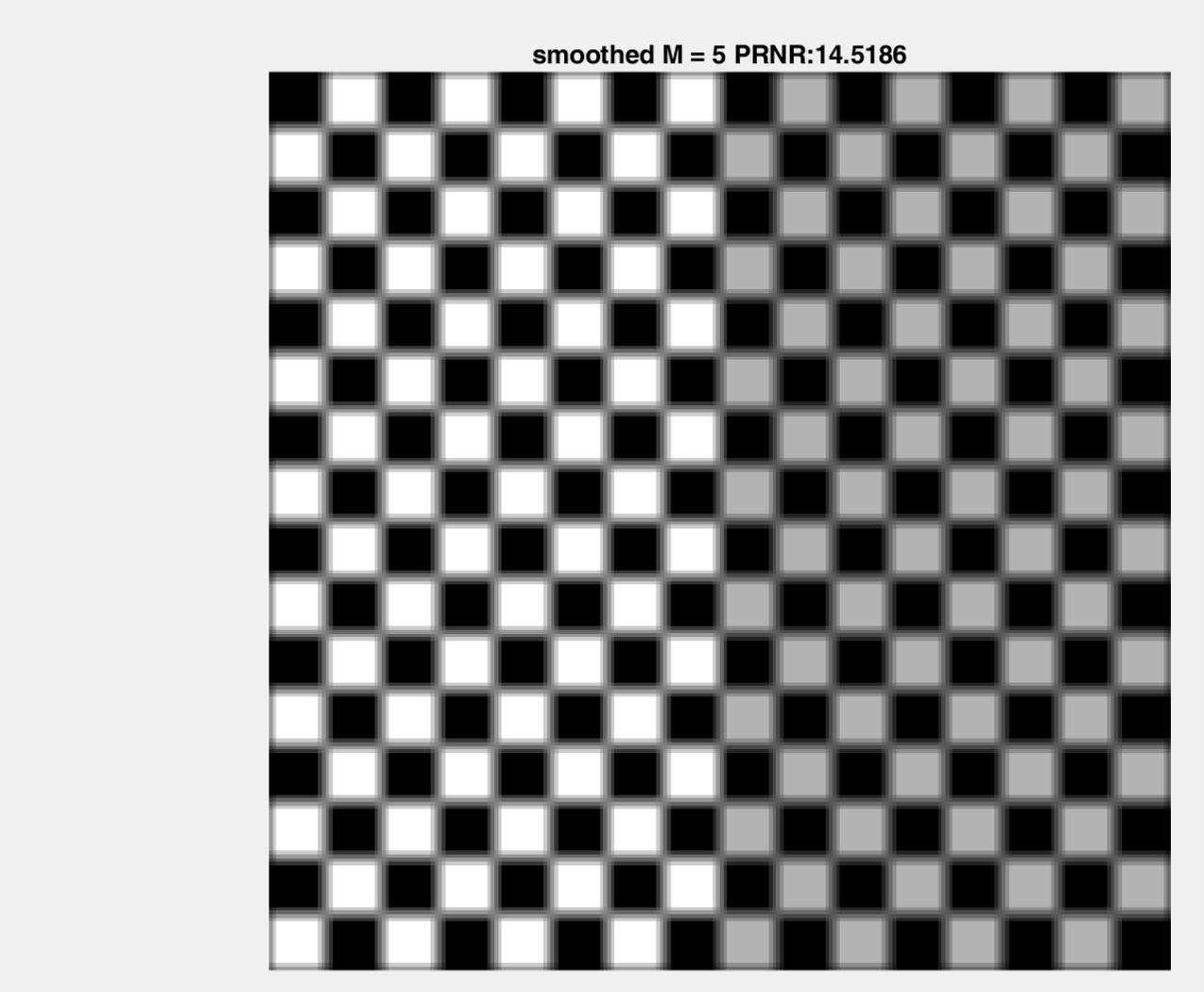
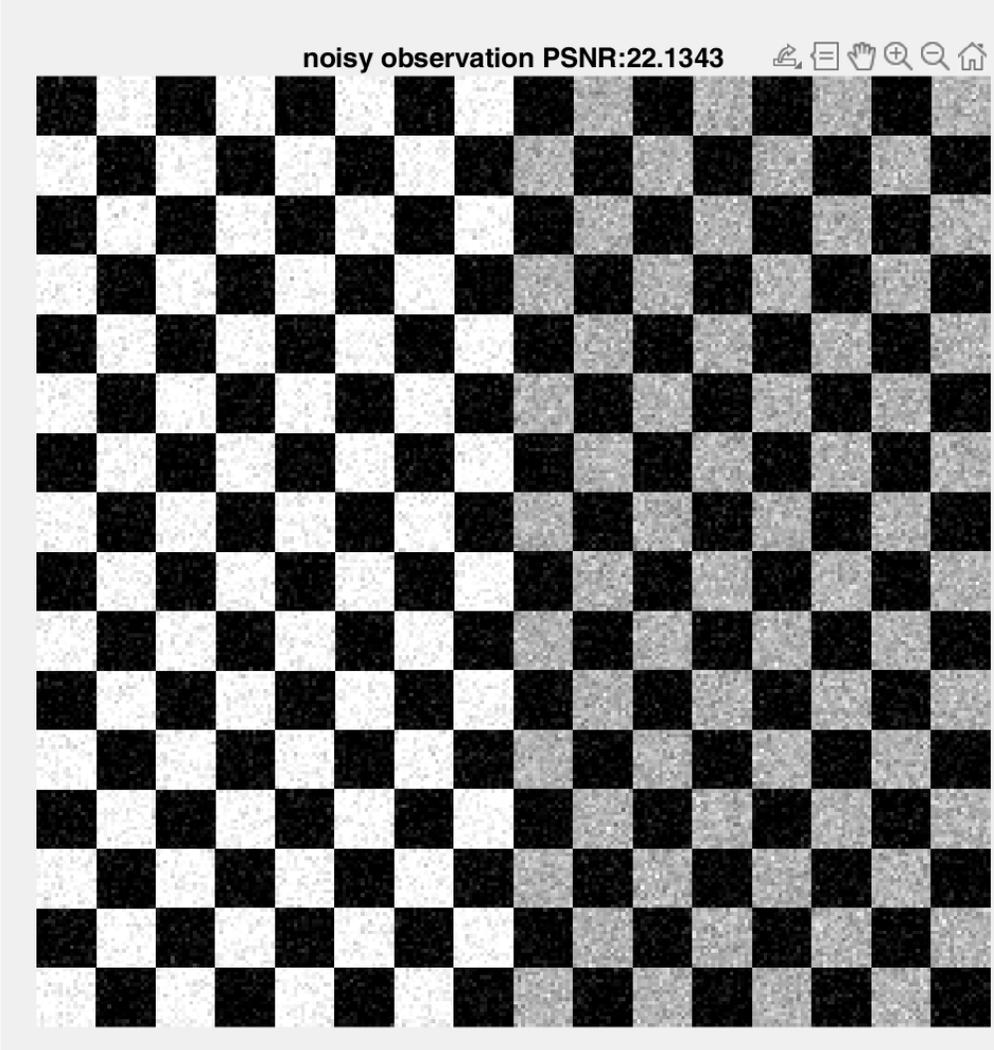
Local Averages Reduce Noise



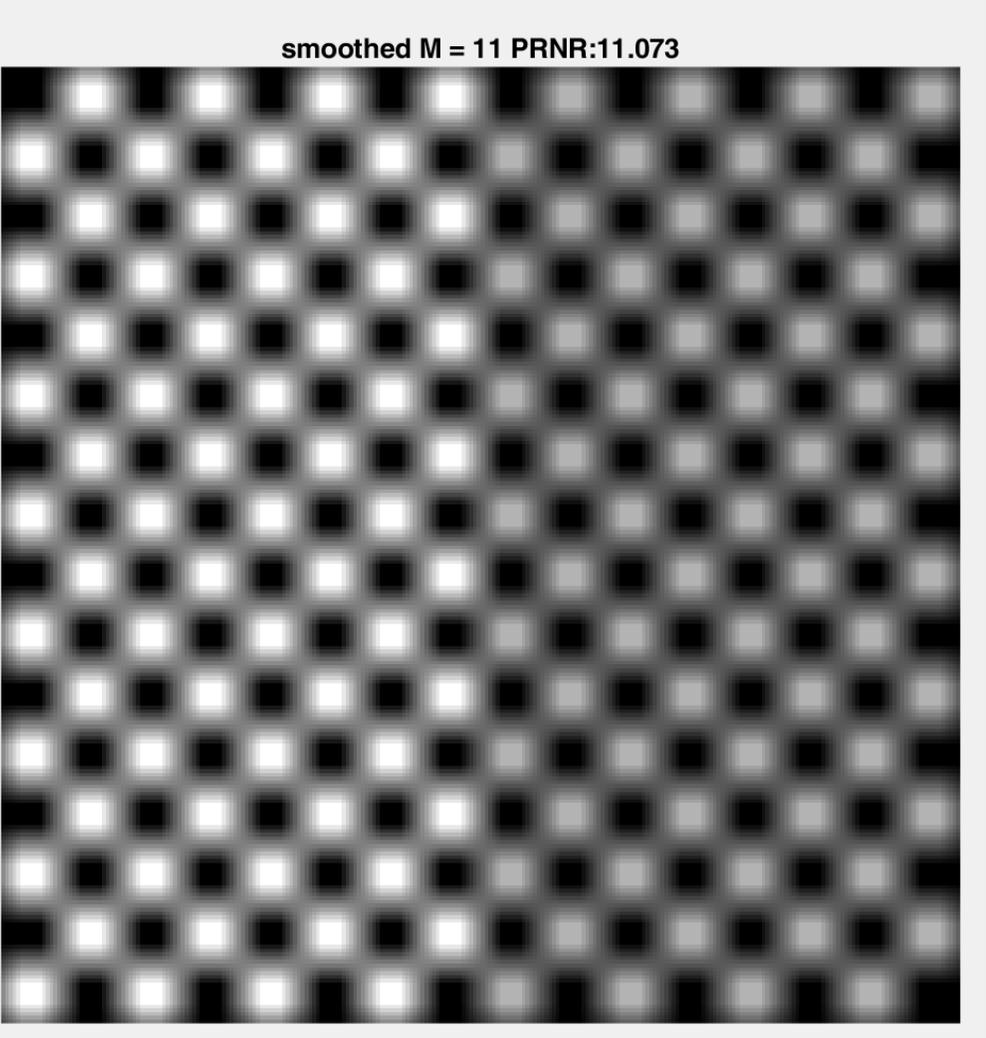
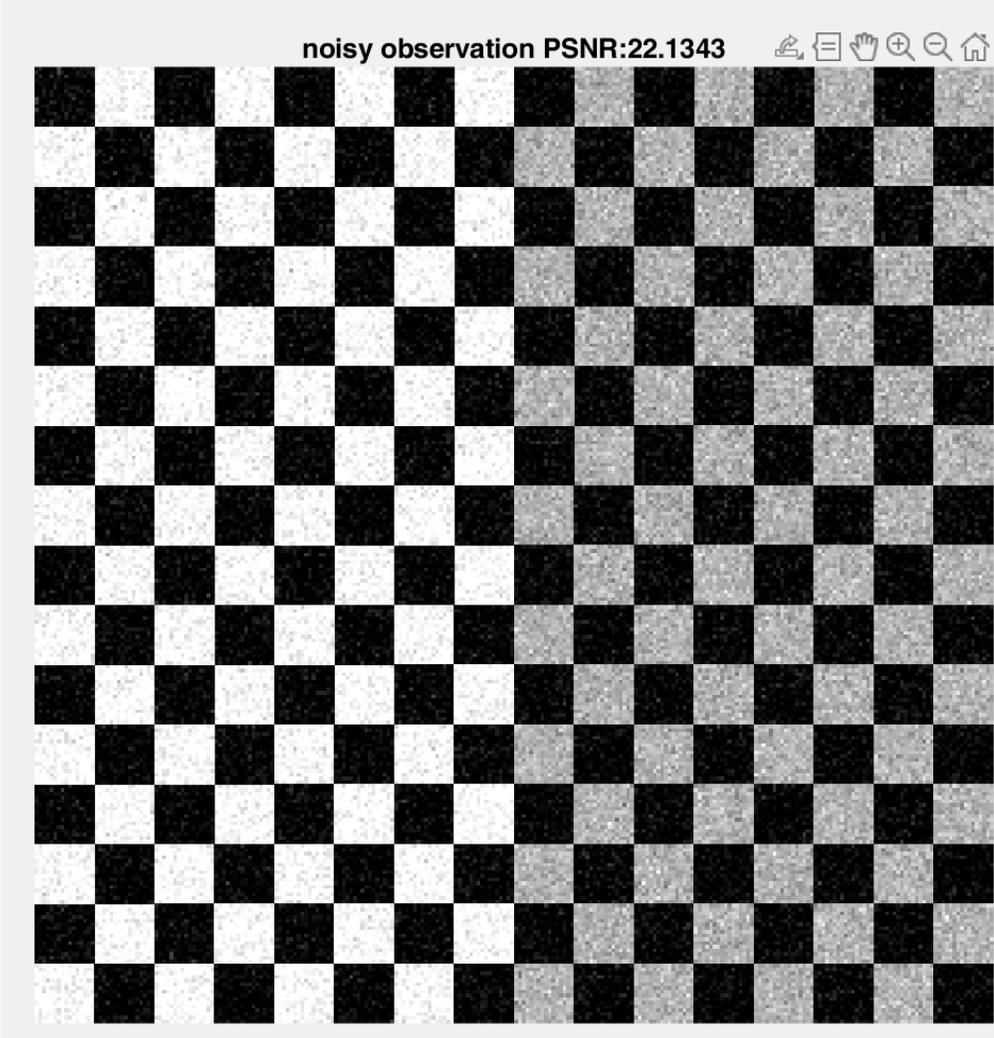
Smoothing by a 3x3 filter



Smoothing by a 5x5 filter

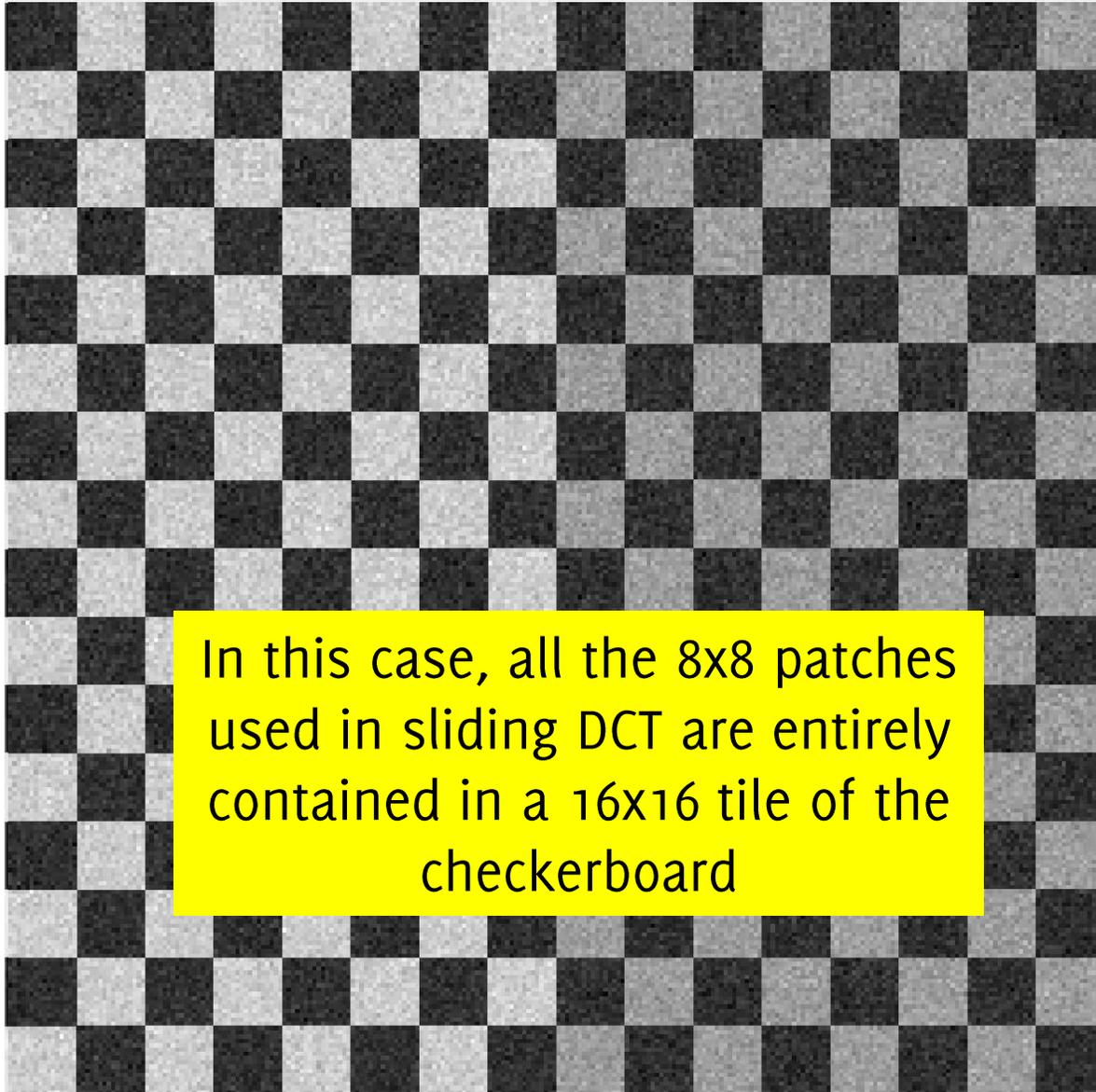


Smoothing by a 5x5 filter

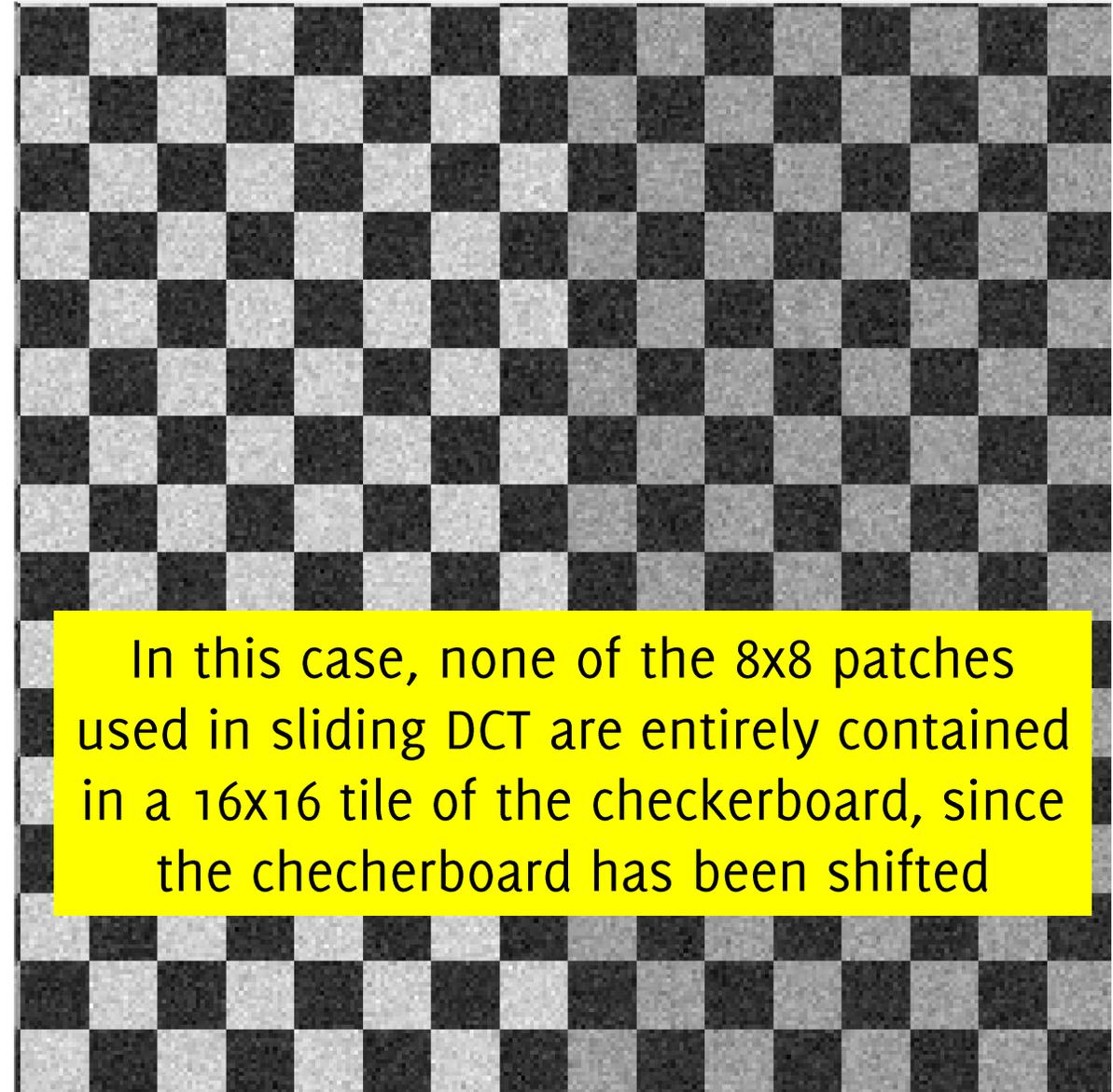


Sliding DCT

Original Checkerboard



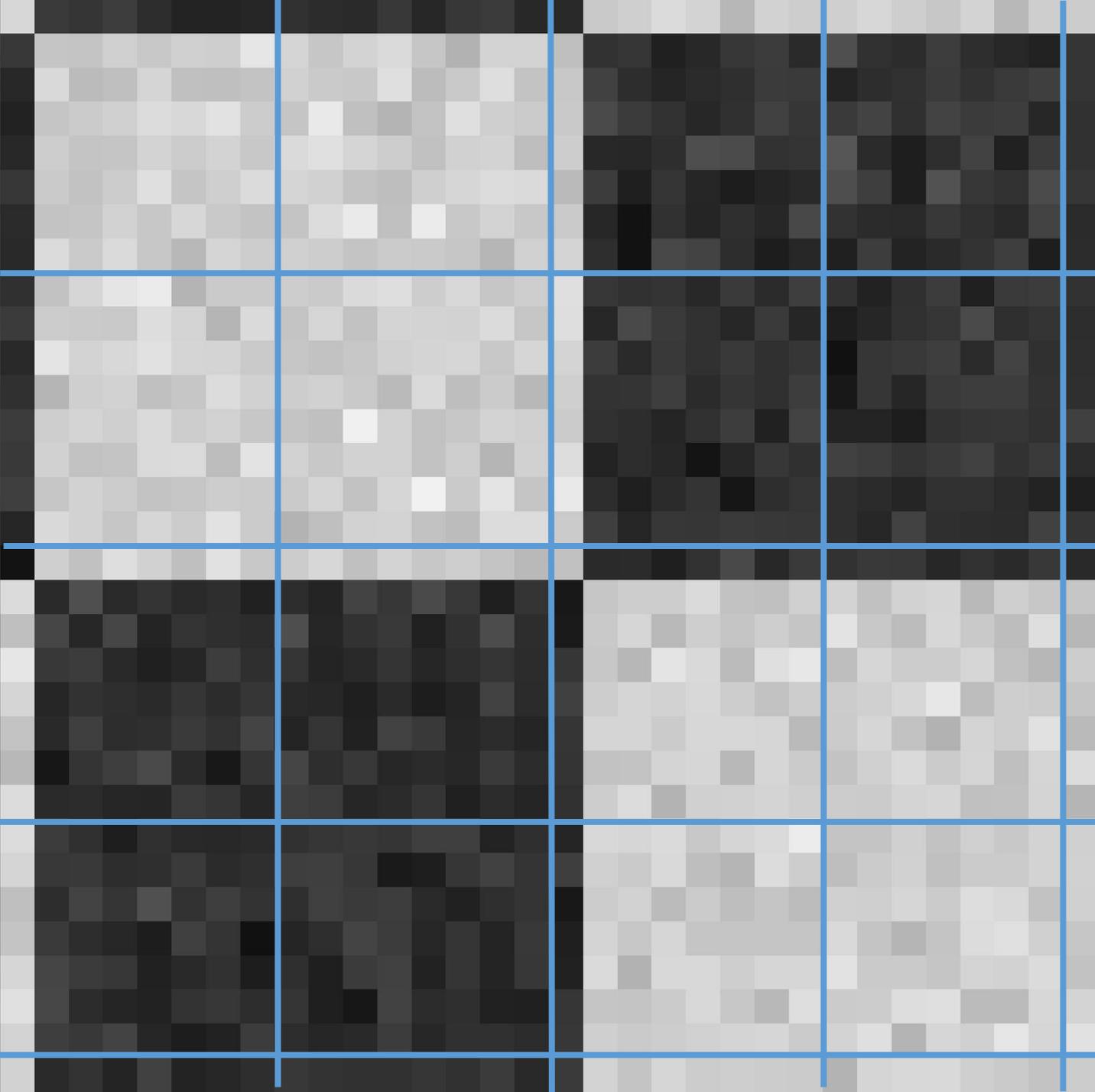
Shift [1 row, 1 col]



Original Checkerboard

Estimated

R : 23.645

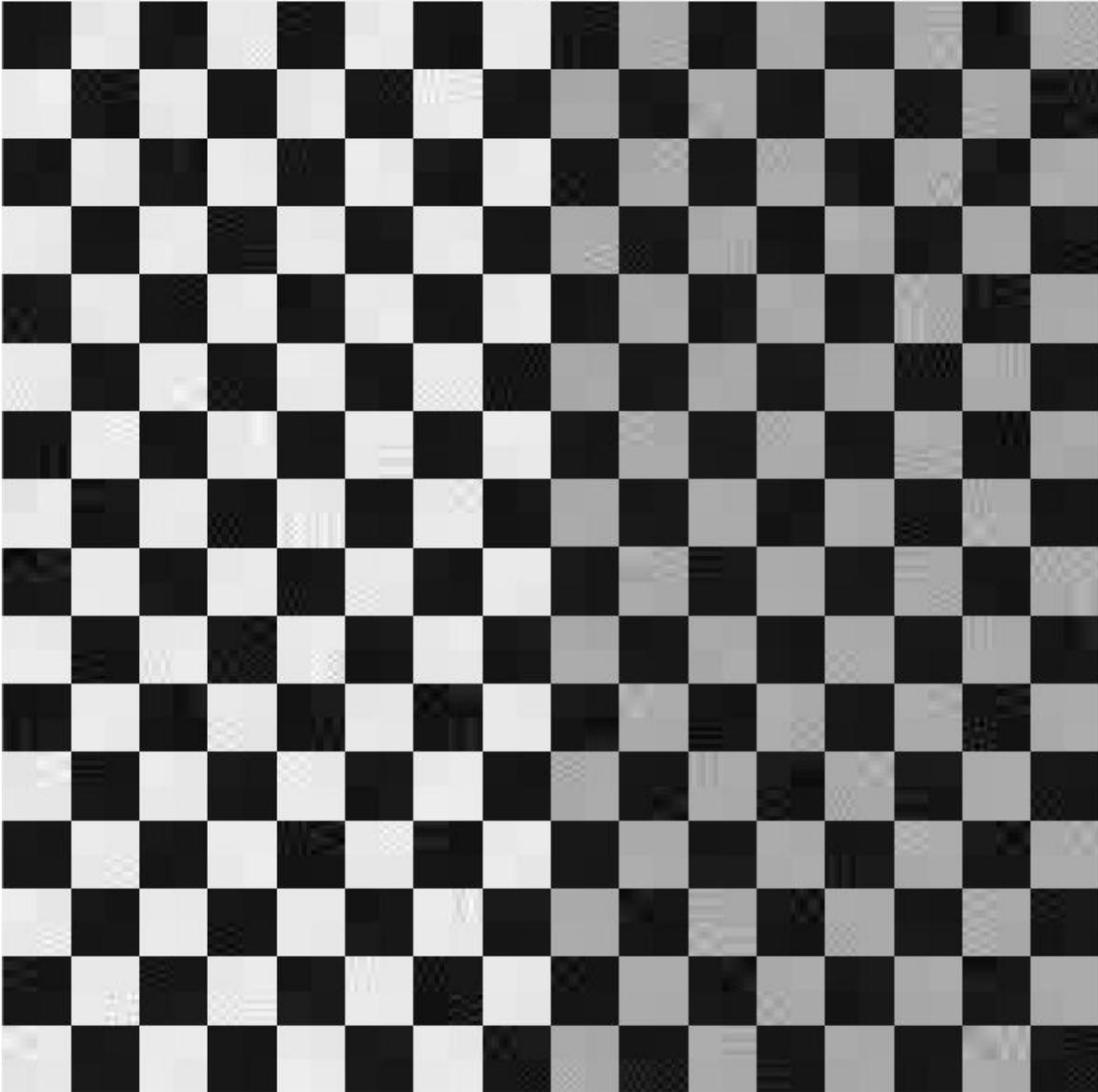


In this case
used in slice
contained in
ch

the 8x8 patches
entirely contained
checkerboard, since
has been shifted

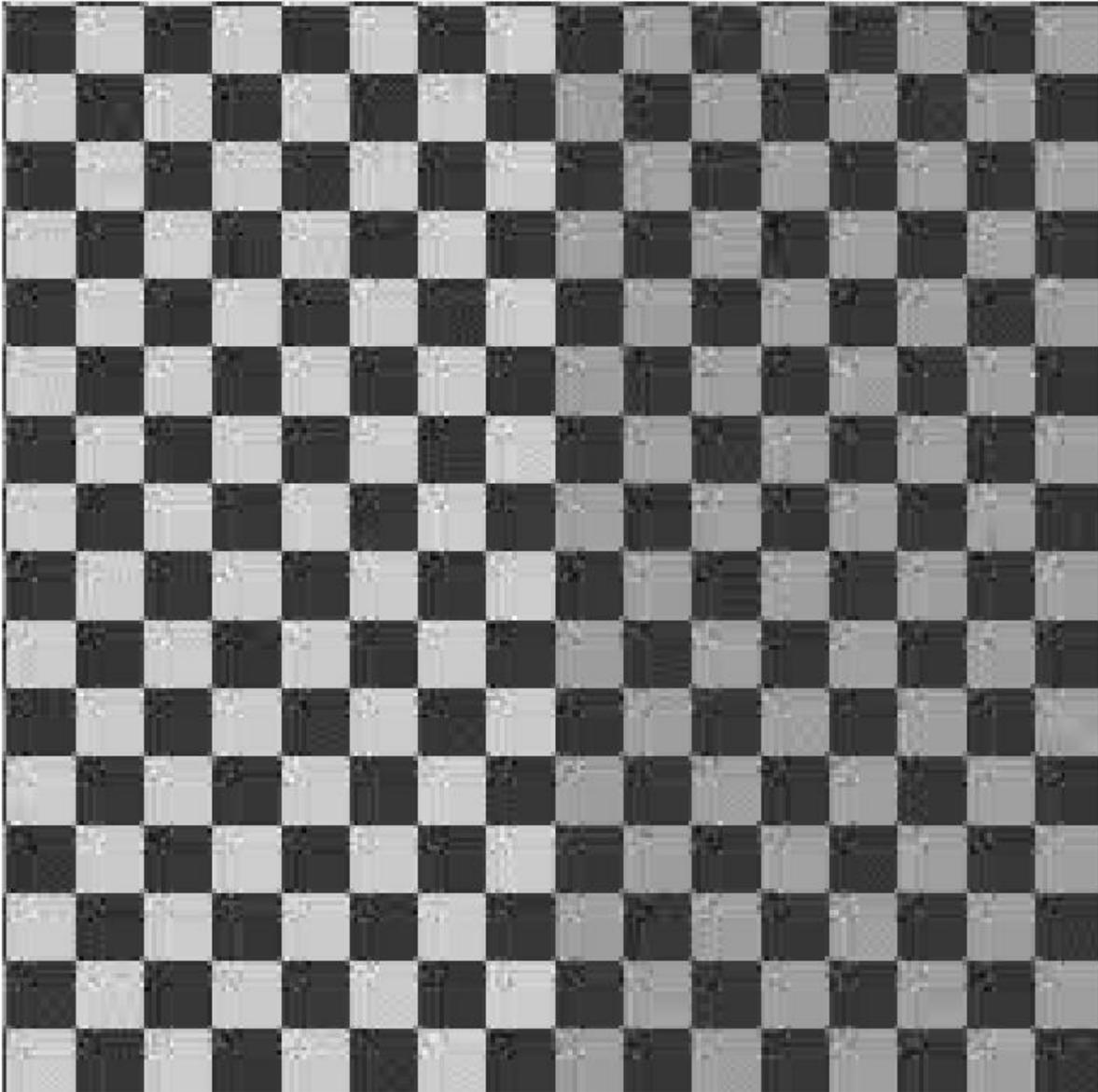
Original Checkerboard

Estimated Image, PSNR : 35.747



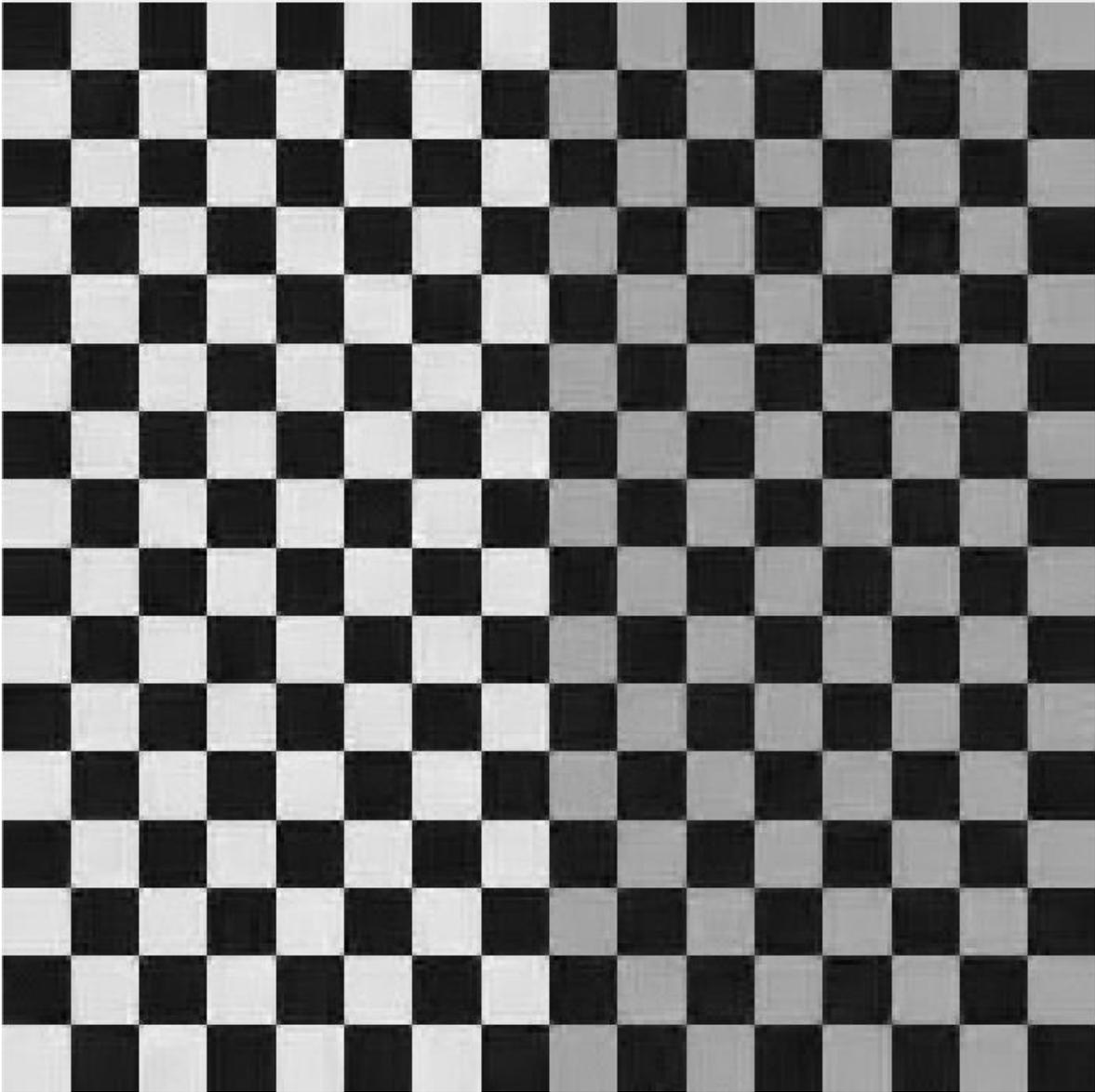
Shift [1 row, 1 col]

Estimated Image, PSNR : 23.645



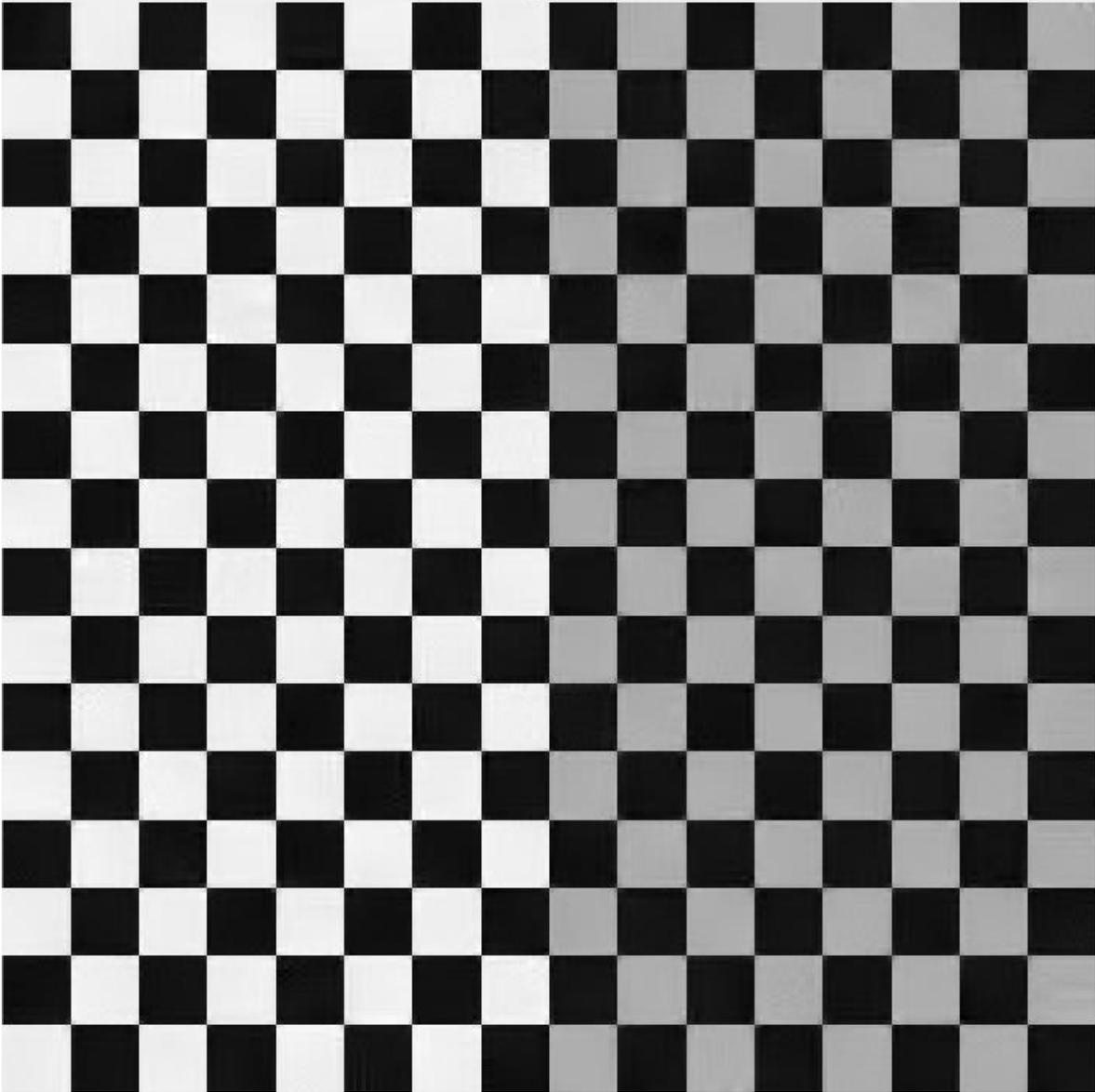
Uniform Weights

Estimated Image, PSNR : 30.582



Sparsity-aware

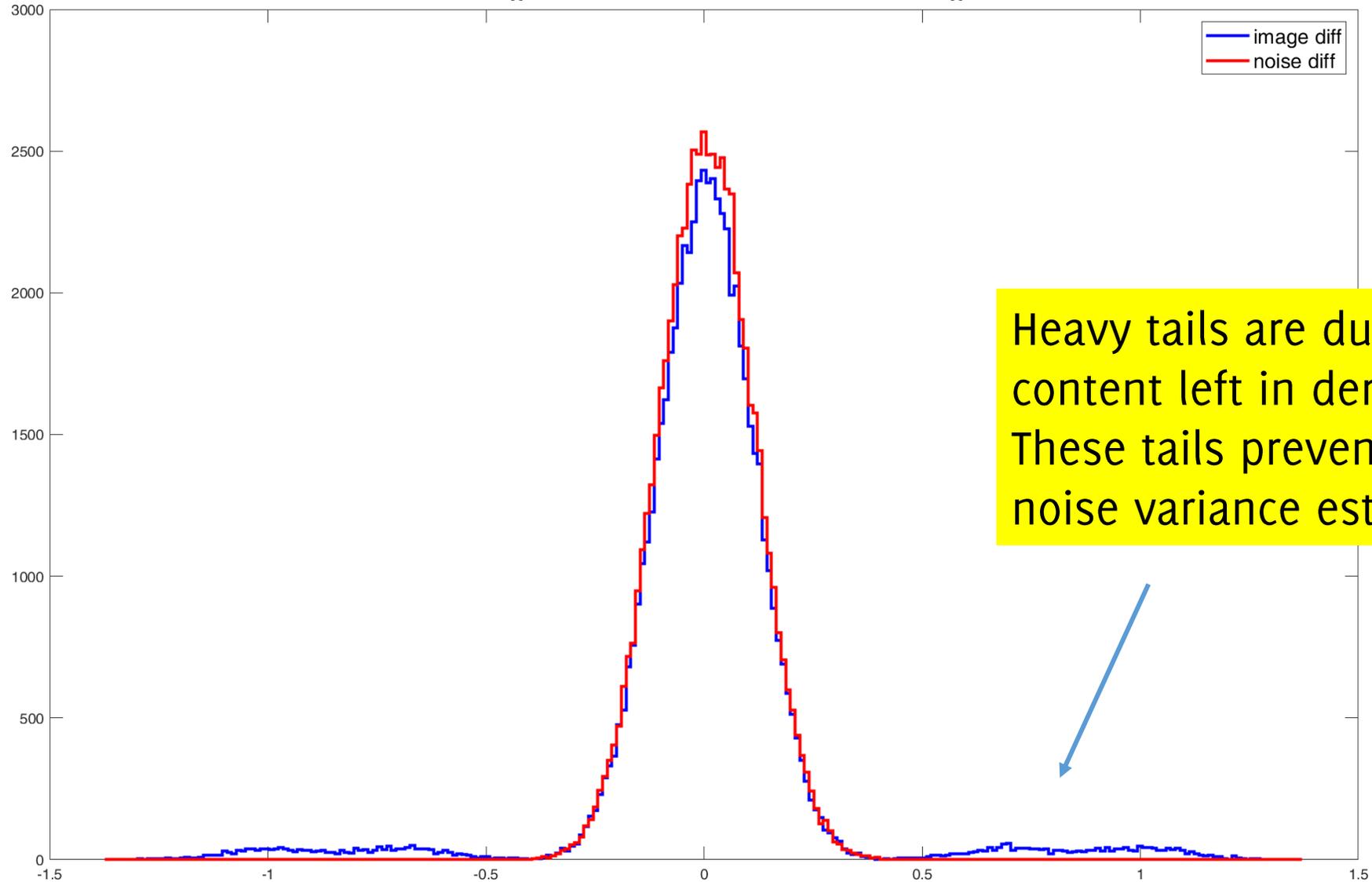
Estimated Image, PSNR : 35.656



Sigma Noise Estimation

Noise Variance Estimation Requires Robust Statistic

σ : 0.078, $\sigma_{h,at}$ (empirical std): 0.169, $\sigma_{h,at}$ (MAD): 0.085



Assignment

1. Implement the sliding-DCT denoising using
 - no aggregation (operate on non-overlapping tiles)
 - aggregation using uniform weights
 - aggregation using weights inversely proportional to patch sparsity in DCT domain.
2. Test the three algorithms on both checkerboard and cameraman image
3. Test how much the choice of the threshold τ influences the denoising performance. Observe the resulting image when:
 - $\tau \ll 3\sigma$
 - $\tau \gg 3\sigma$

This is very important to understand how important is the choice of the threshold

Original image



Noisy image, PSNR = 22.09



HT Estimate, PSNR = 29.15



Wiener Estimate, PSNR = 29.41

