Patch Modeling and JPEG

Mathematical Models and Methods for Image Processing
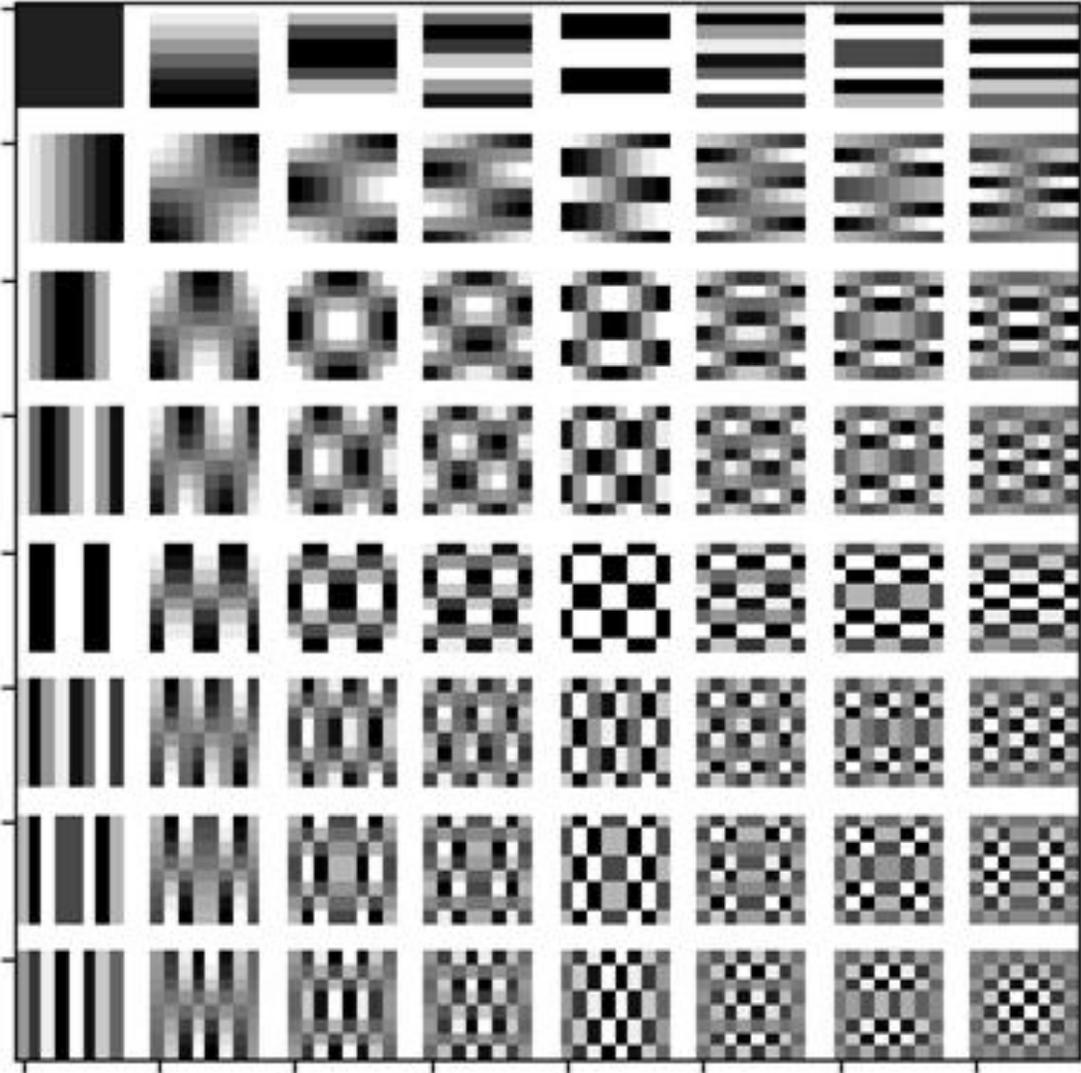
Stefano Bertolasi

https://boracchi.faculty.polimi.it/teaching/MMMIP.htm
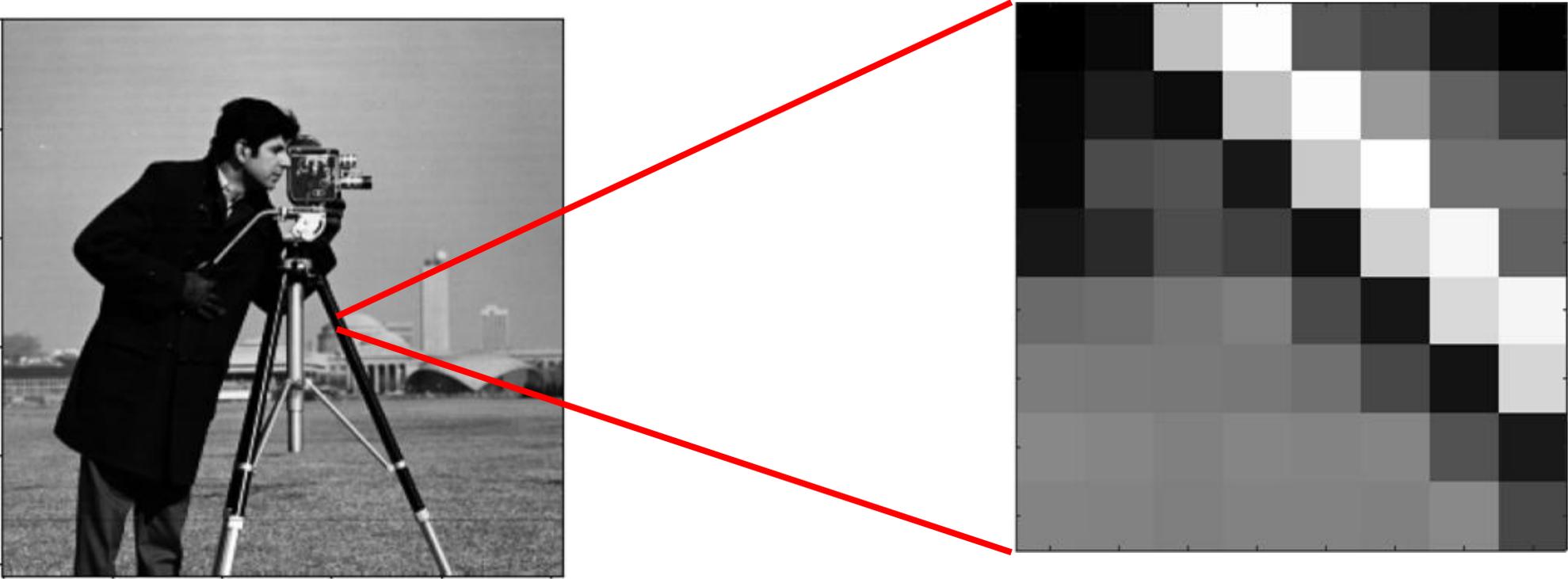
February 26th 2026

2D DCT

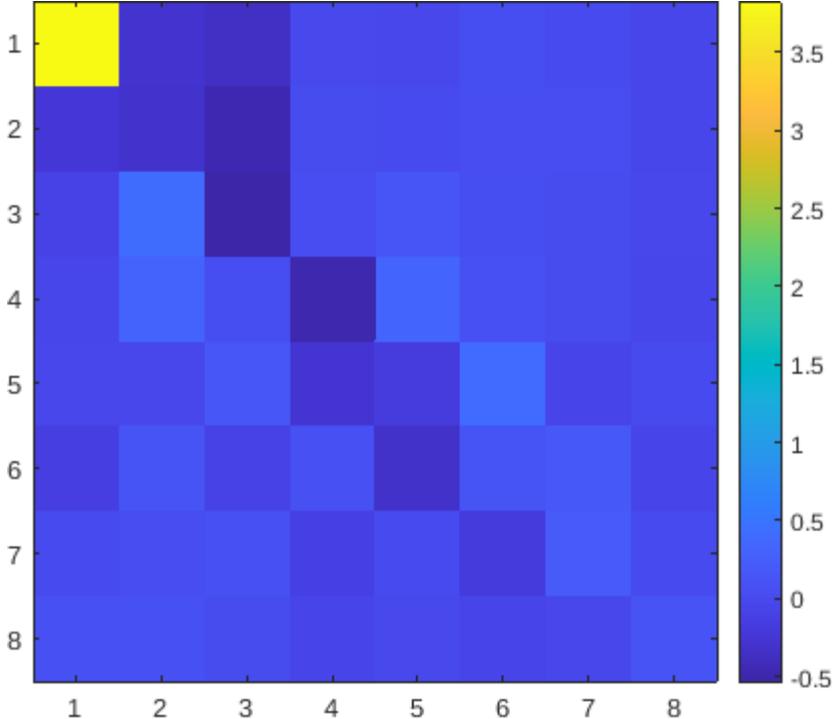# How the atoms in the 2D DCT dictionary look like

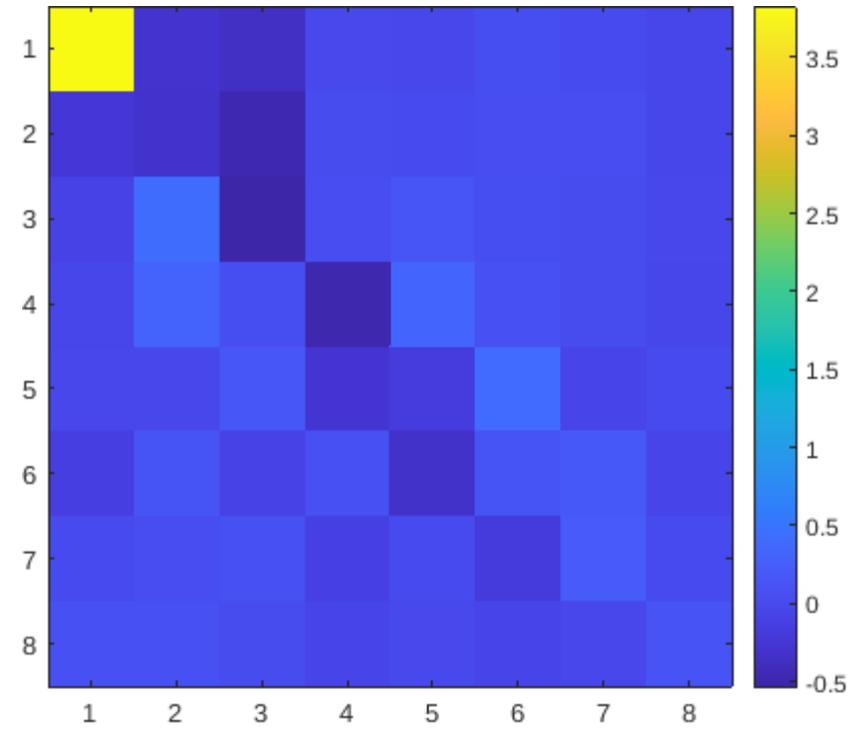# The JPEG Compression

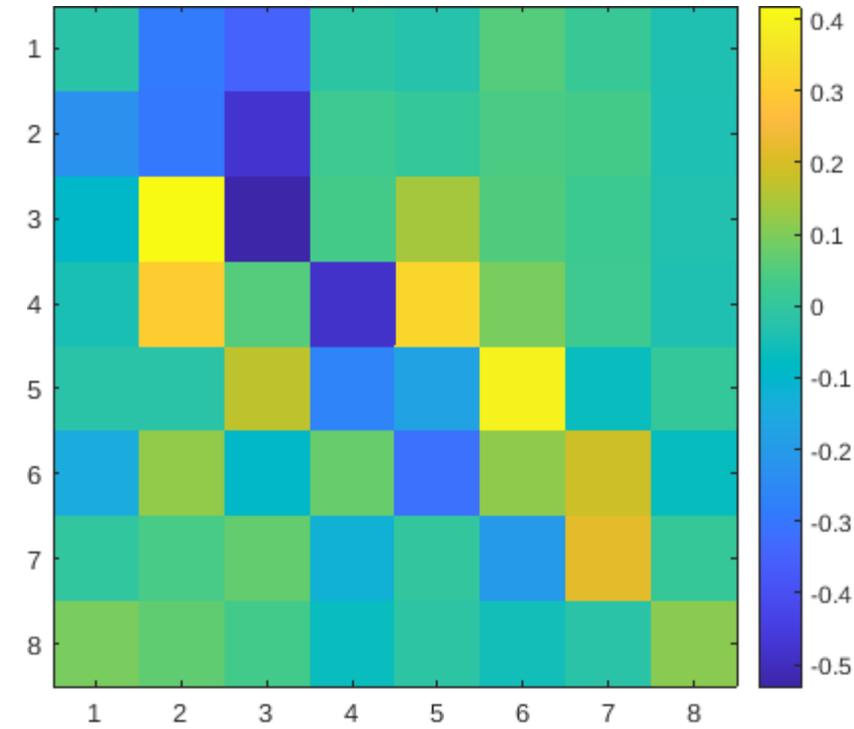# Let's extract a 8x8 patch from an image

2D DCT of the patch

2D DCT of the patch
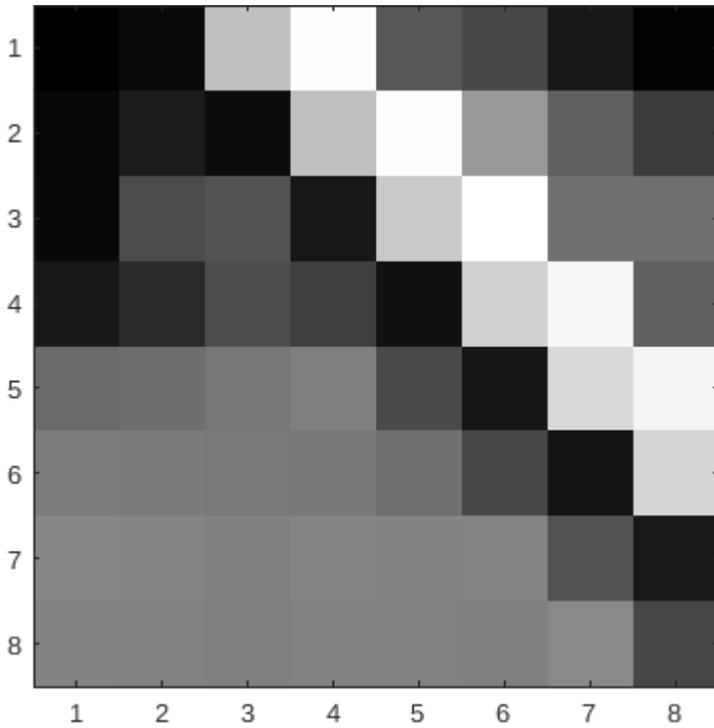

With DC

Without DC

A lot of coefficients are closed to o!

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Reconstructed patch
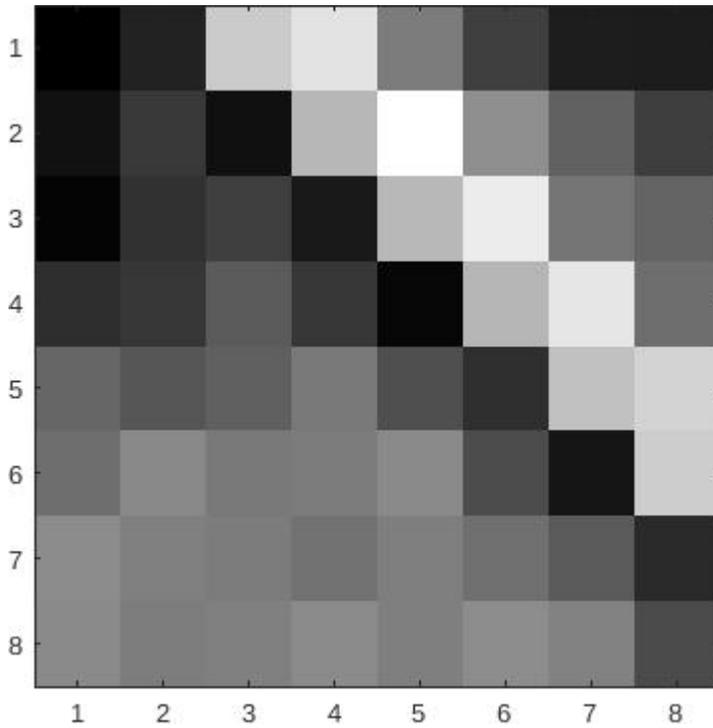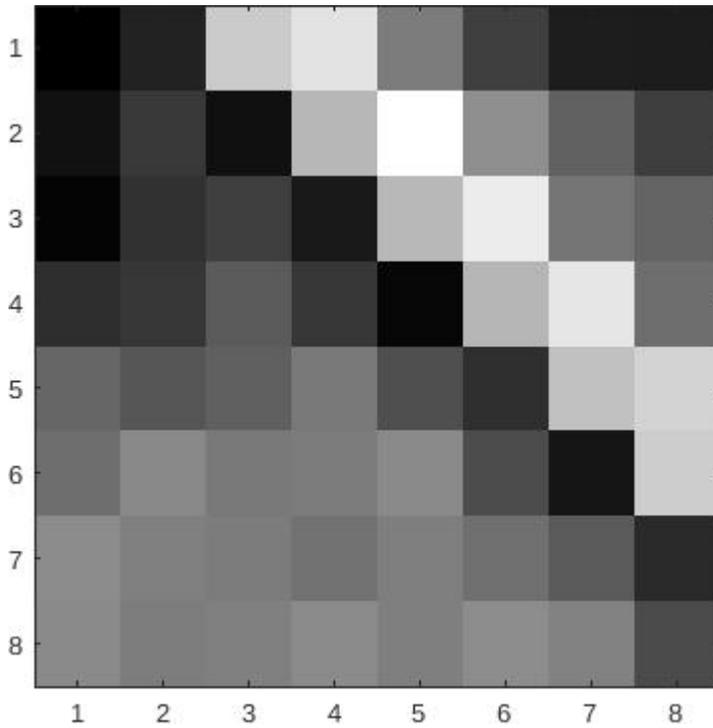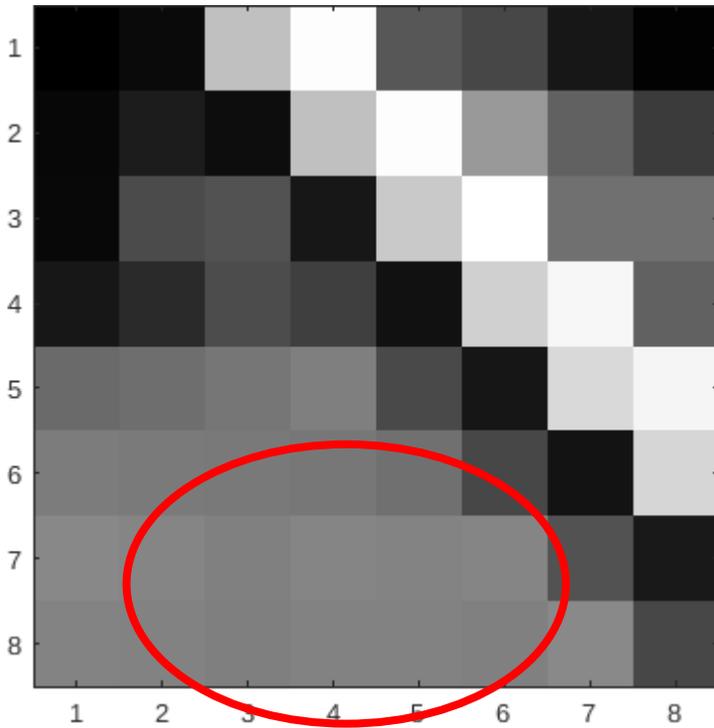
Original Patch

Reconstructed Patch

Reconstructed patch

Original Patch

Reconstructed Patch

Smoothing in this area

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Assignments

Last Time Assignment: Generate the Basis

- Generate the DCT basis according to the following formula (DCT type II). Remember tha the k-th atom of the DCT basis in dimension M is defines as

$$DCT_k(n) = c_k \cos\left(k\pi\frac{2n+1}{2M}\right) \quad n, k = 0, ..., M-1,$$

$$c_0 = \sqrt{1/M}, \qquad c_k = \sqrt{2/M} \text{ for } k \neq 0.$$

- How can we use the function `dct` and its inverse `idct` to define the DCT matrix?

First Assignment: 2D DCT dictionary

- Generate the 2D DCT dictionary using the `dct2` and `idct2` functions
  - Use this dictionary to compute the representation of a patch
- Generate the 1D DCT dictionary using the `dct` and `idct` functions
  - Use this dictionary to compute the separable 2D DCT of the same patch
- Verify that the coefficients of the two representations are the same
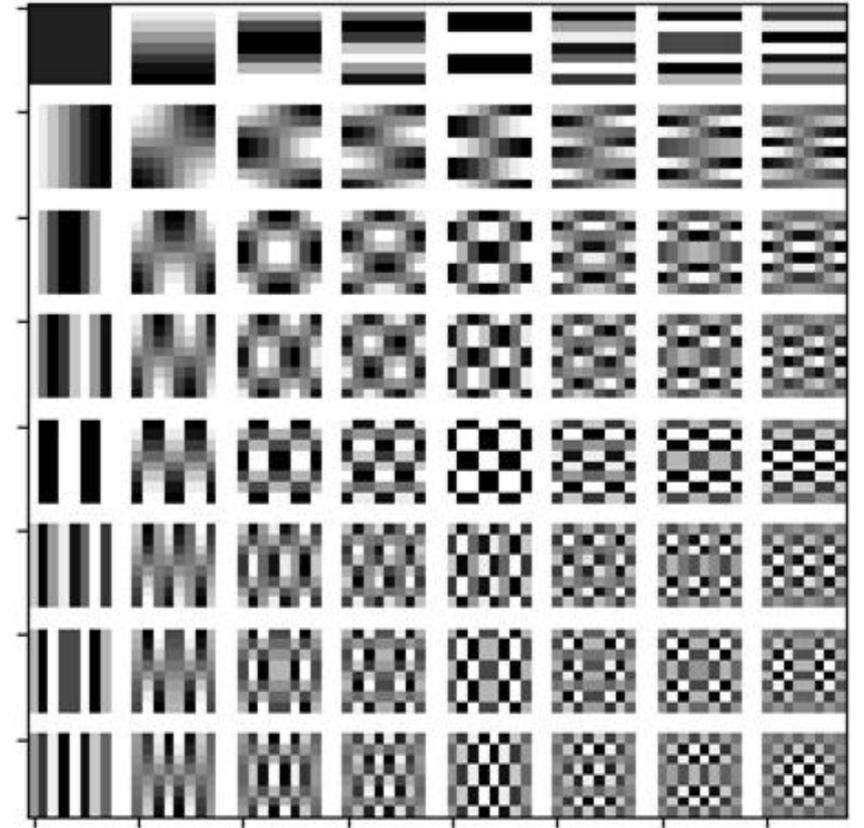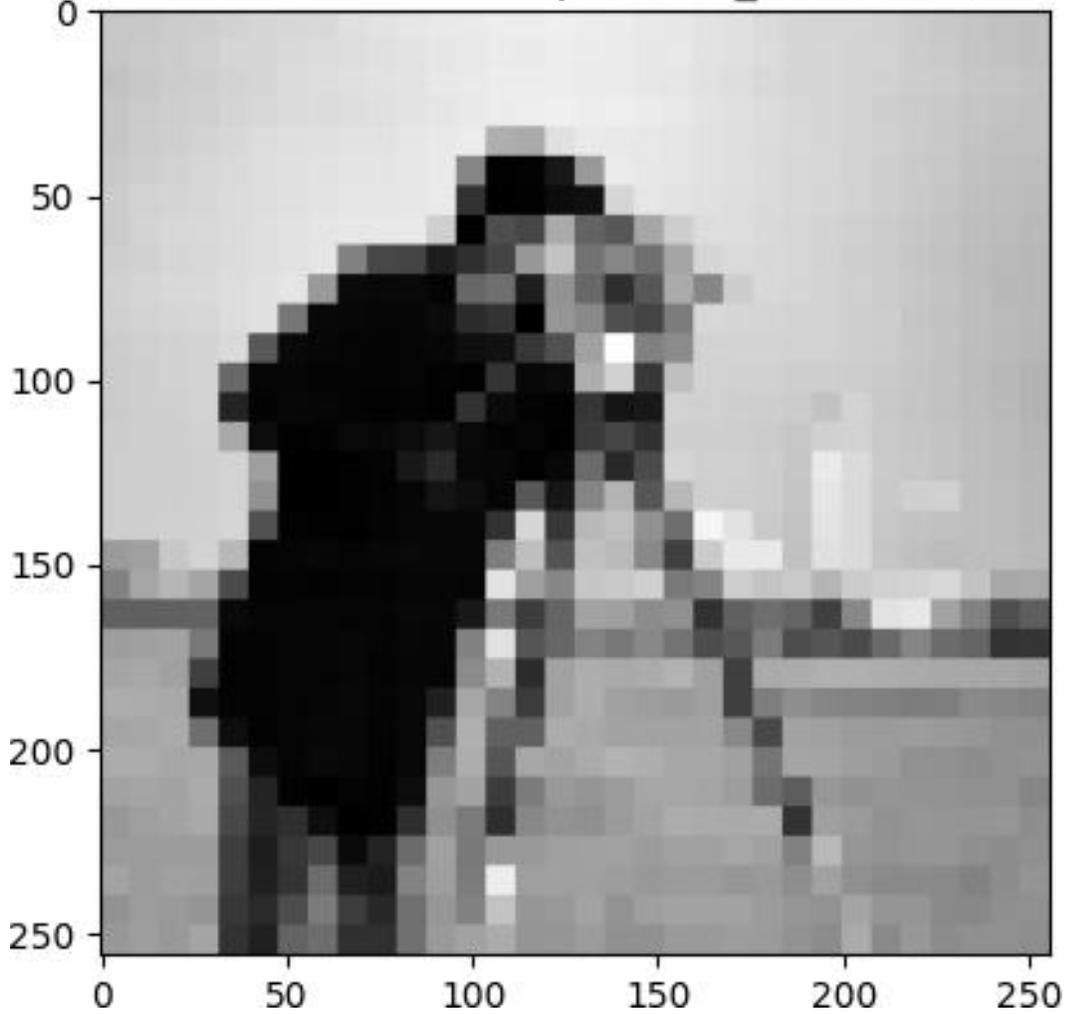
Second Assignment: JPEG Compression

- Implement the JPEG compression algorithm

- Compute the PSNR of the compressed image

- Compute the compression ratio

- Try different thresholds: how the thresholds affect the compression?

- Run the JPEG compression on the test image and try different threshold. What can you observe?
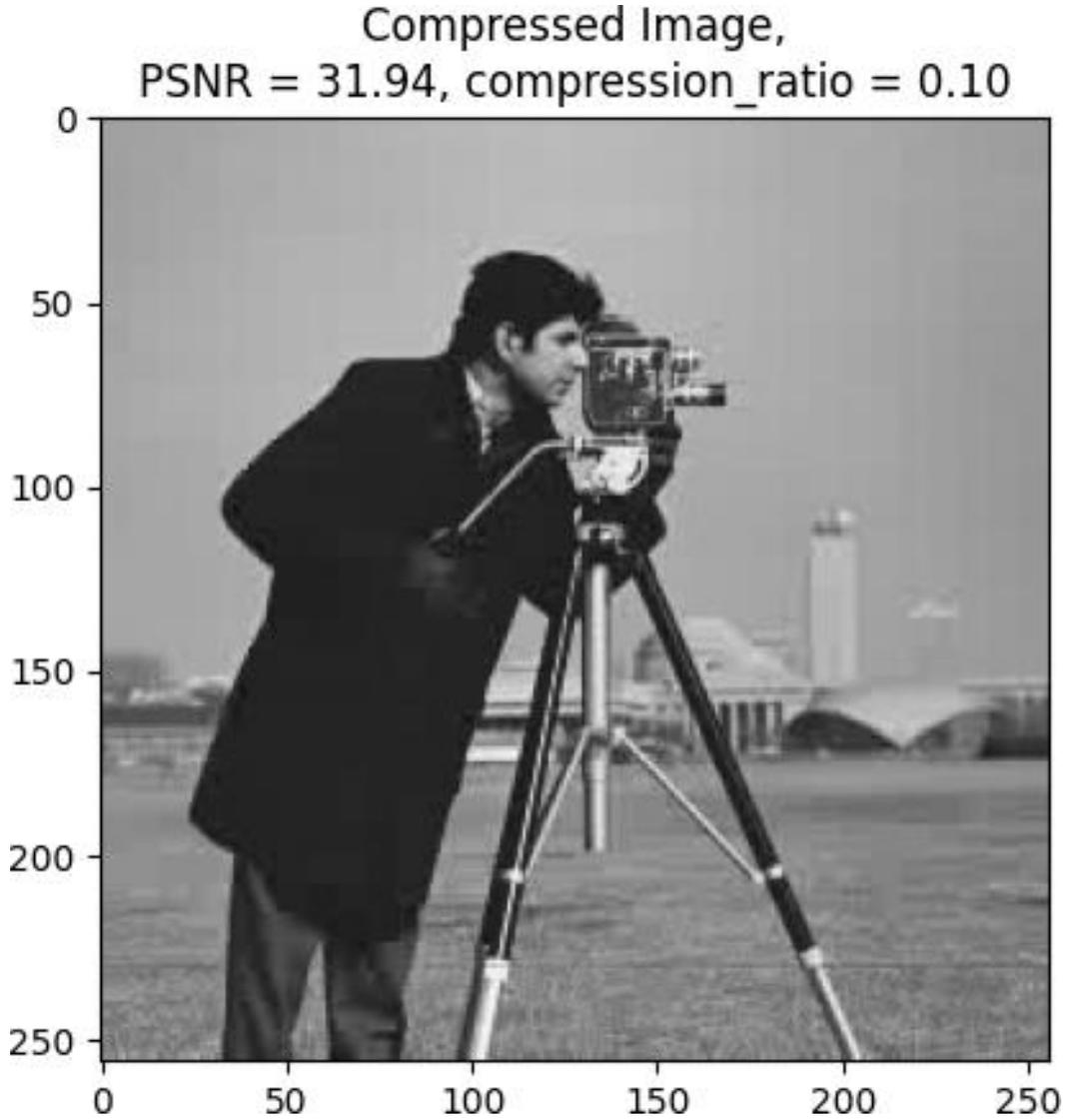
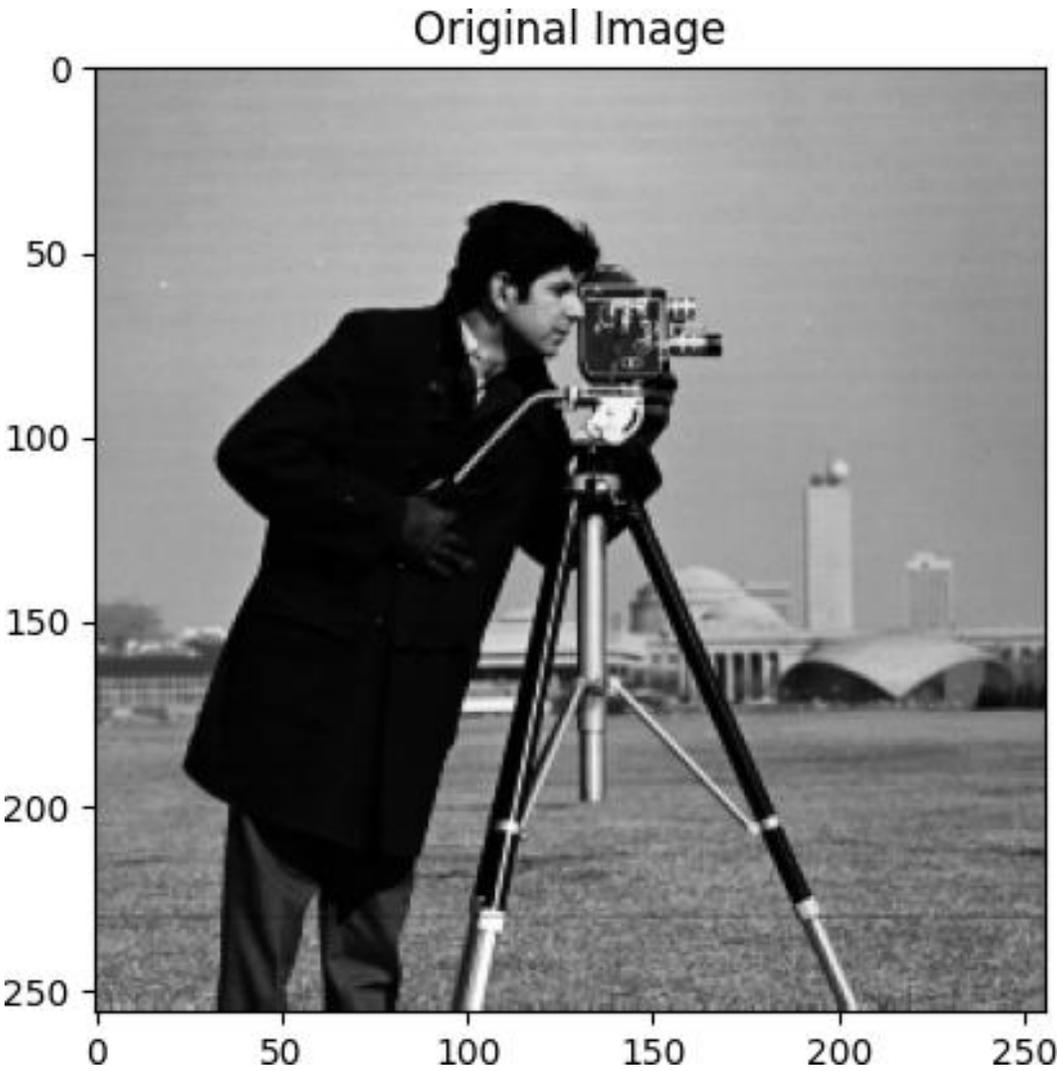# Second Assignment: JPEG Compression

Original Image
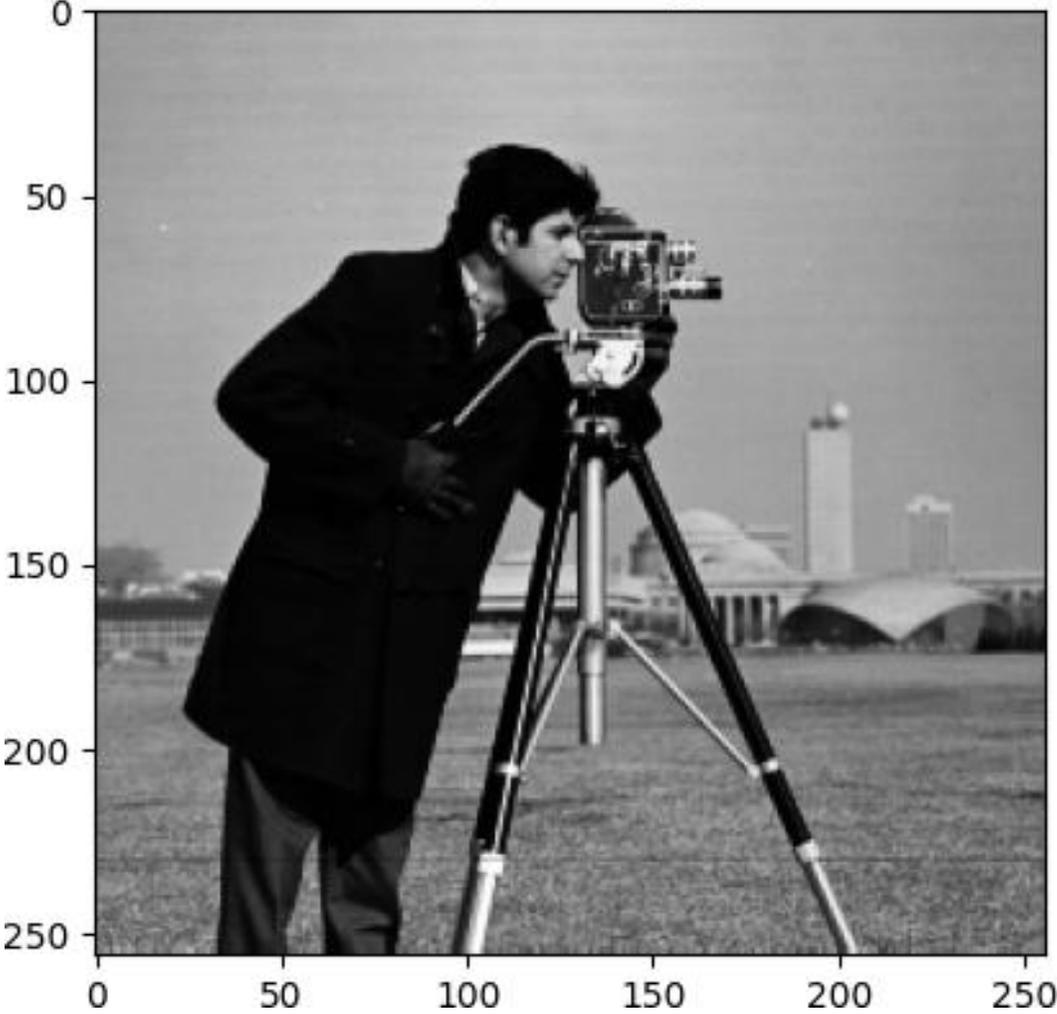
Compressed Image,
PSNR = 19.48, compression_ratio = 0.02

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Second Assignment: JPEG Compression

Original Image

Compressed Image,
PSNR = 31.94, compression_ratio = 0.10

Second Assignment: JPEG Compression

Compressed Image,
PSNR = 31.94, compression_ratio = 0.10

S. Bertolasi