

Local Polynomial Approximation

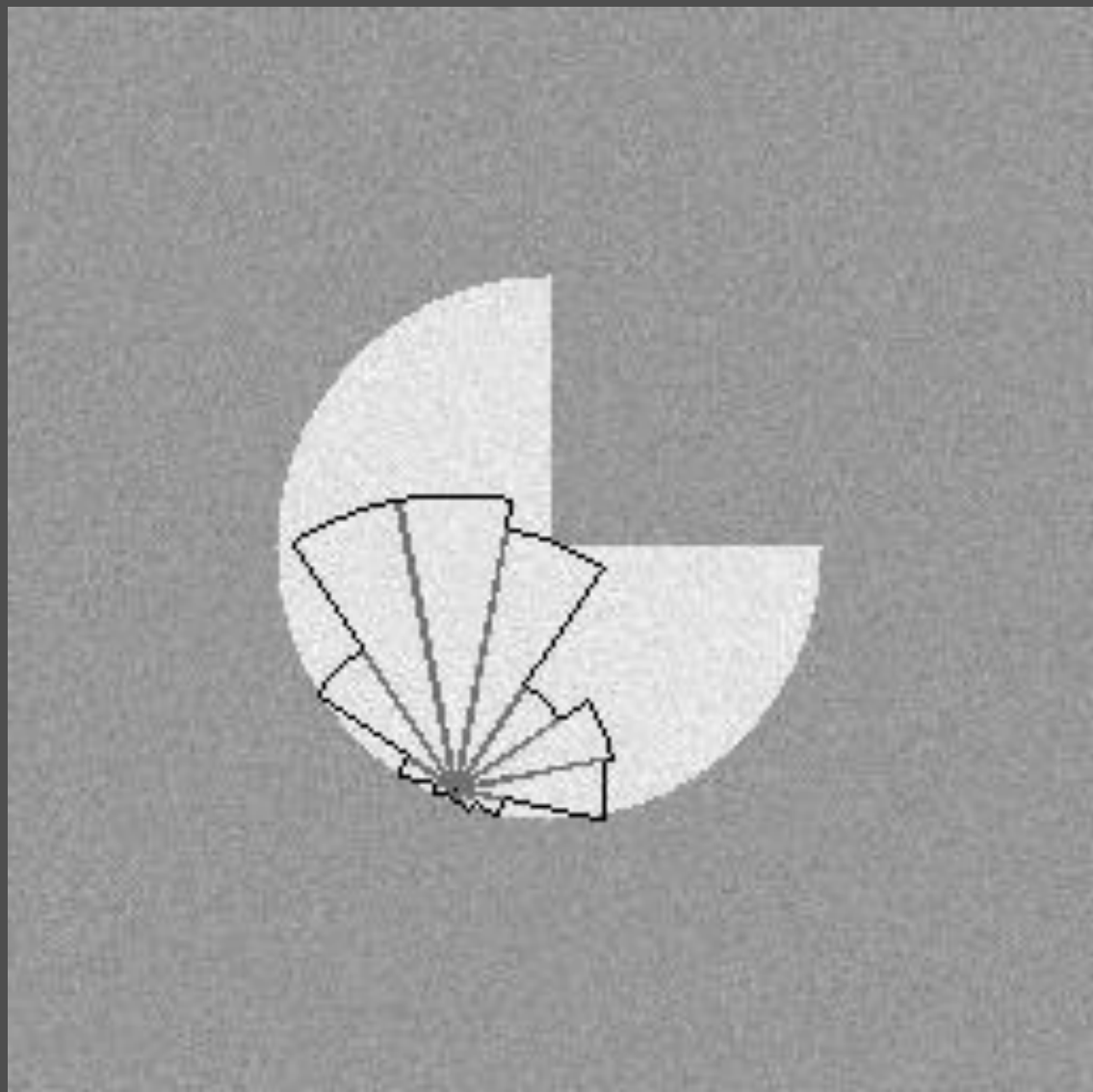
Mathematical Models and Methods for Image Processing

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https://webpages.tuni.fi/foi/Present/Anis_Web.html



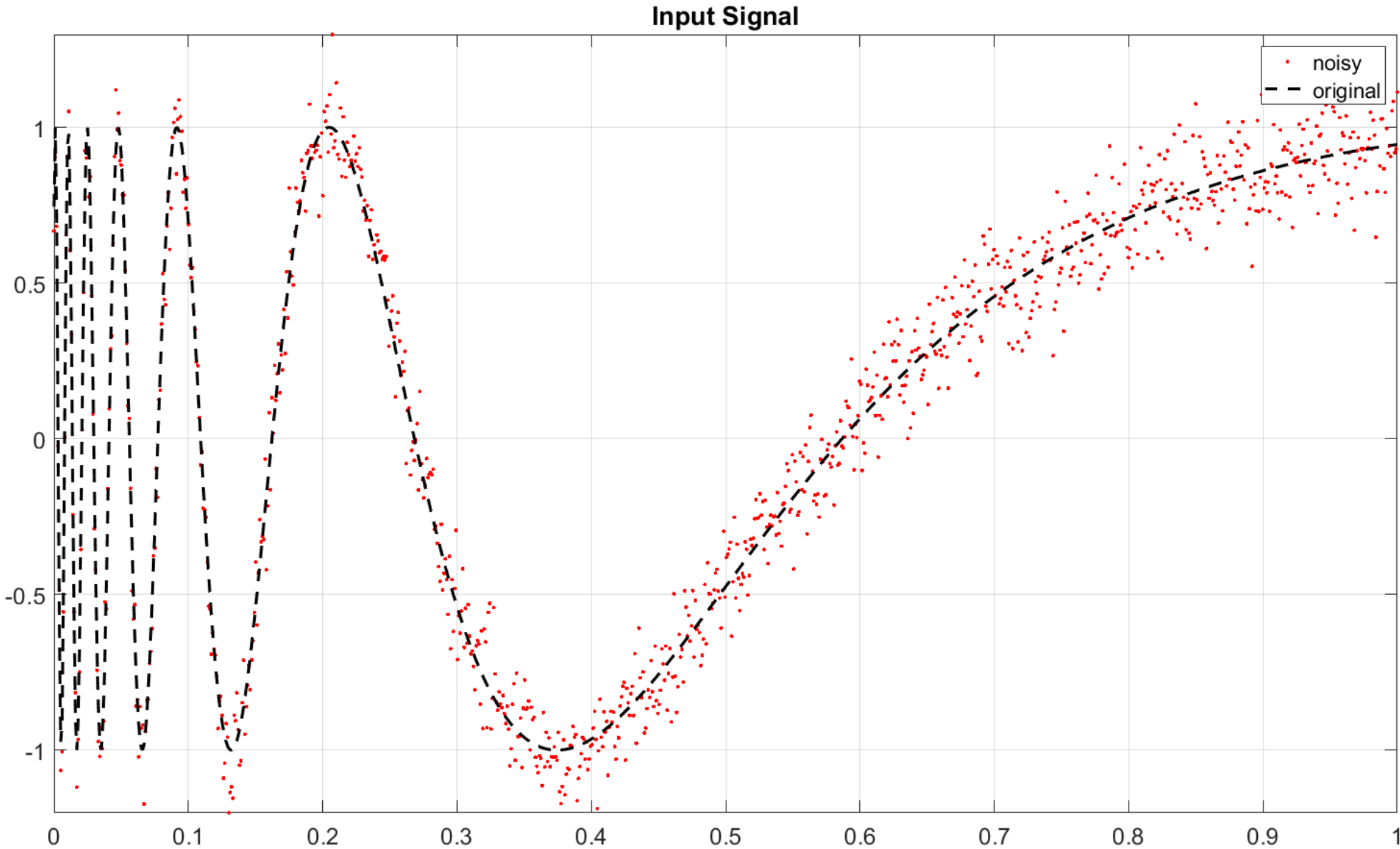
Assignment 1: LPA Kernels

A. Foi, *Anisotropic nonparametric image processing: theory, algorithms and applications*, Ph.D. Thesis, Dip. di Matematica, Politecnico di Milano, April 2005.

Lez21_A_LPA

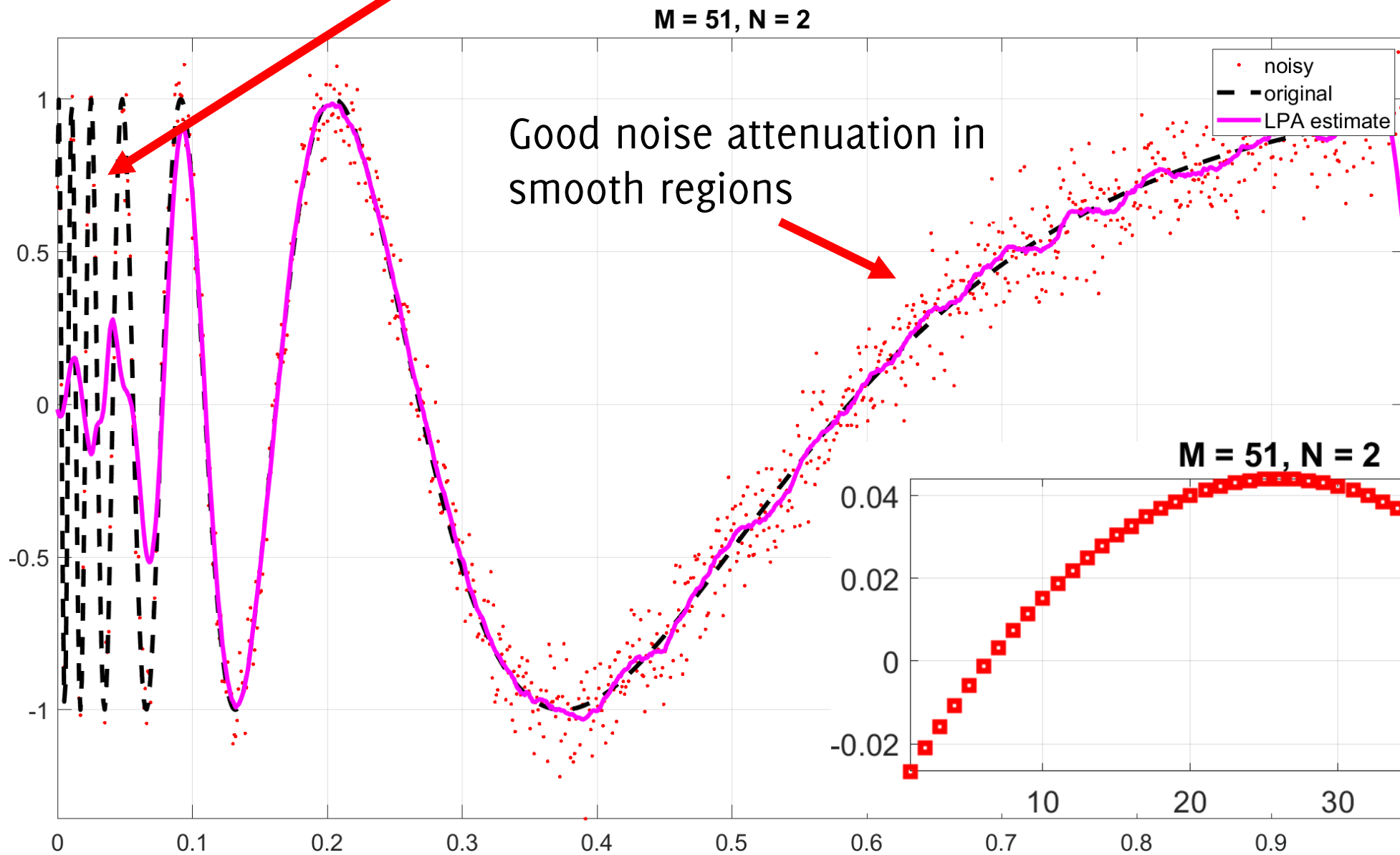
Define the LPA filters for a given polynomial order N and over a fixed support M to perform regression over noisy signals

Noisy Signal



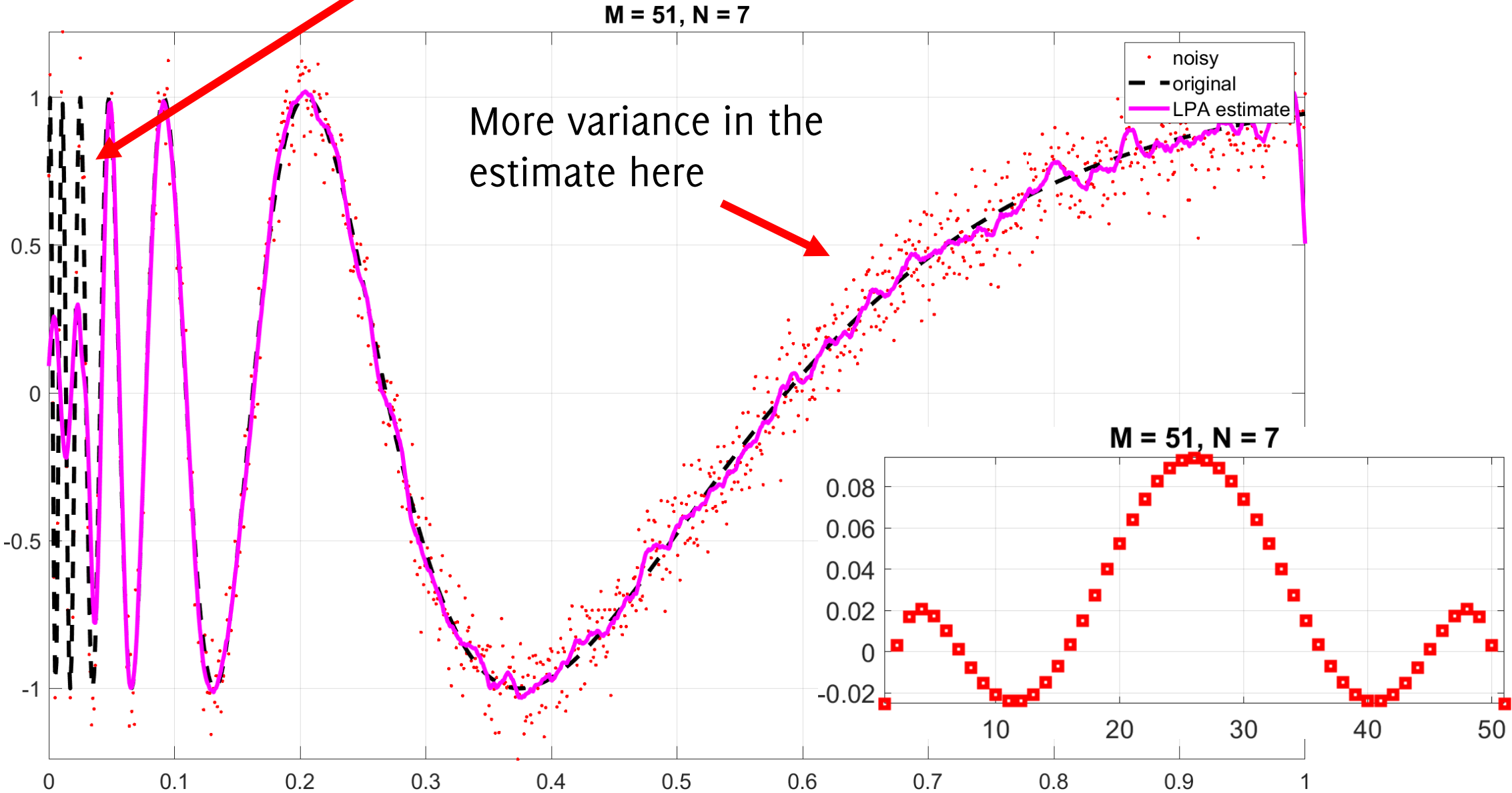
Large M , small N

Biased estimate when the signal frequency is high



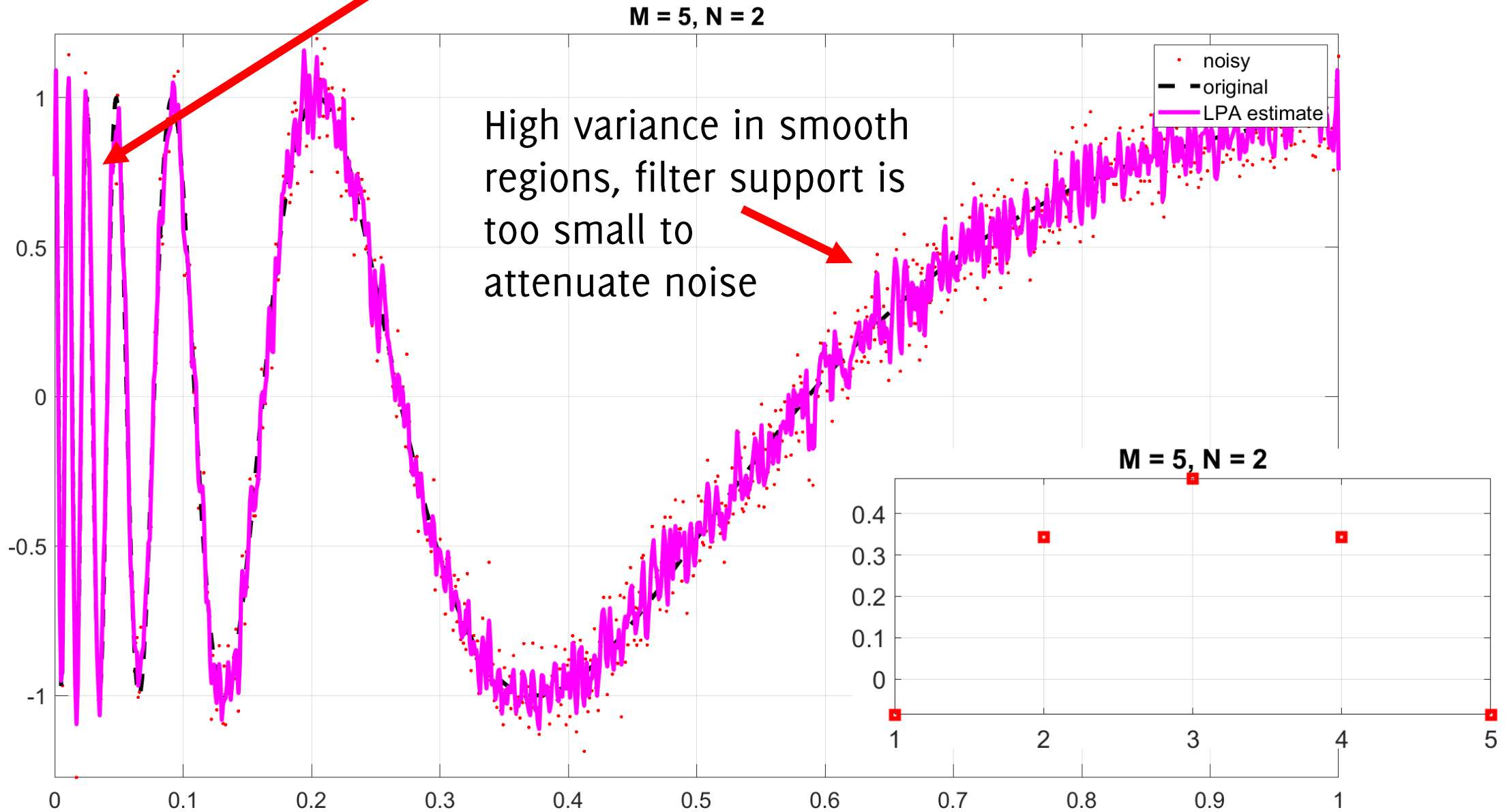
Large M , large N

Lower bias than before when the signal frequency is high



small M , small N

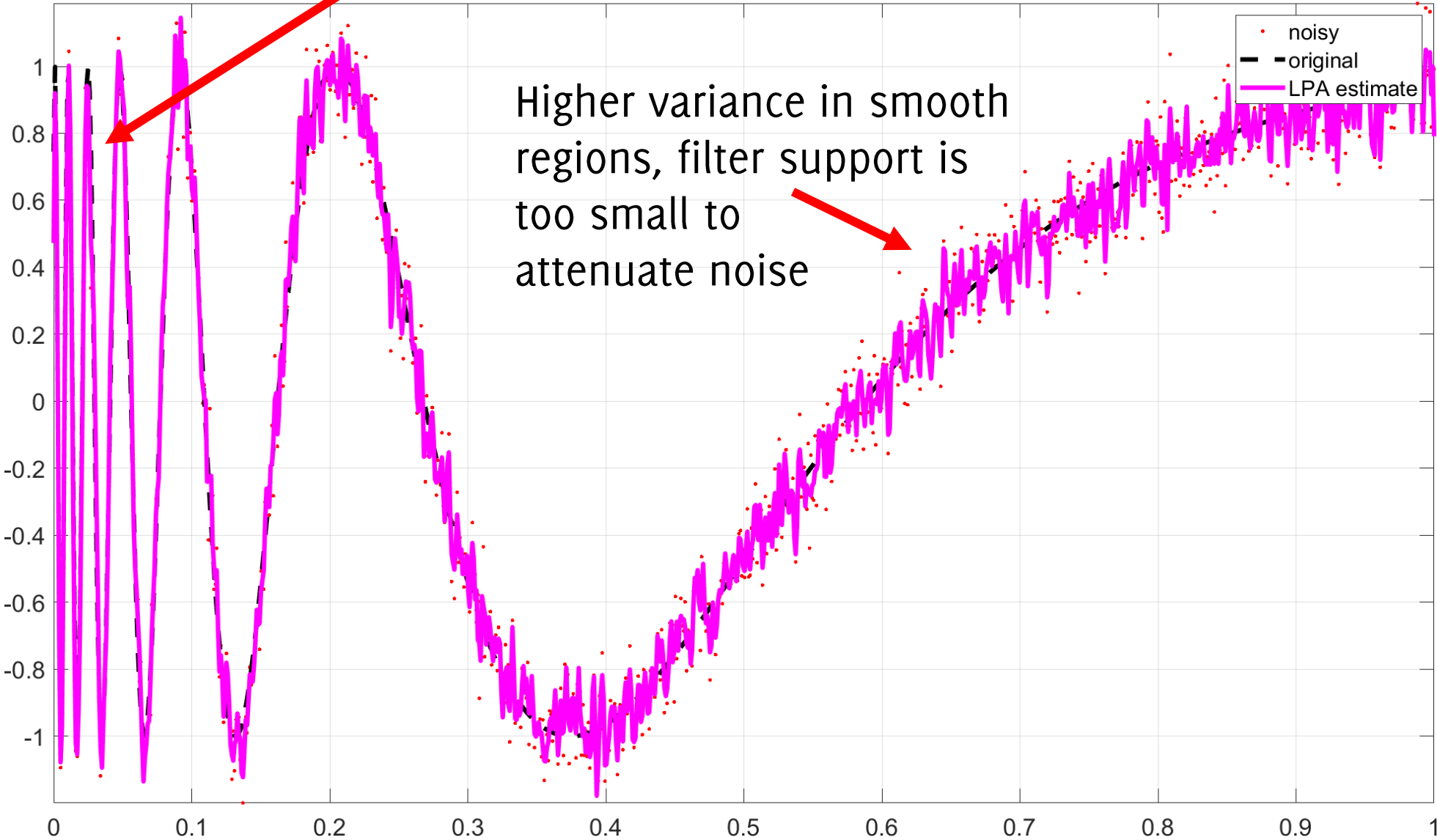
Lower bias than before when the signal frequency is high



small M , comparable N

Lower bias than before when the signal frequency is high

$M = 7, N = 5$



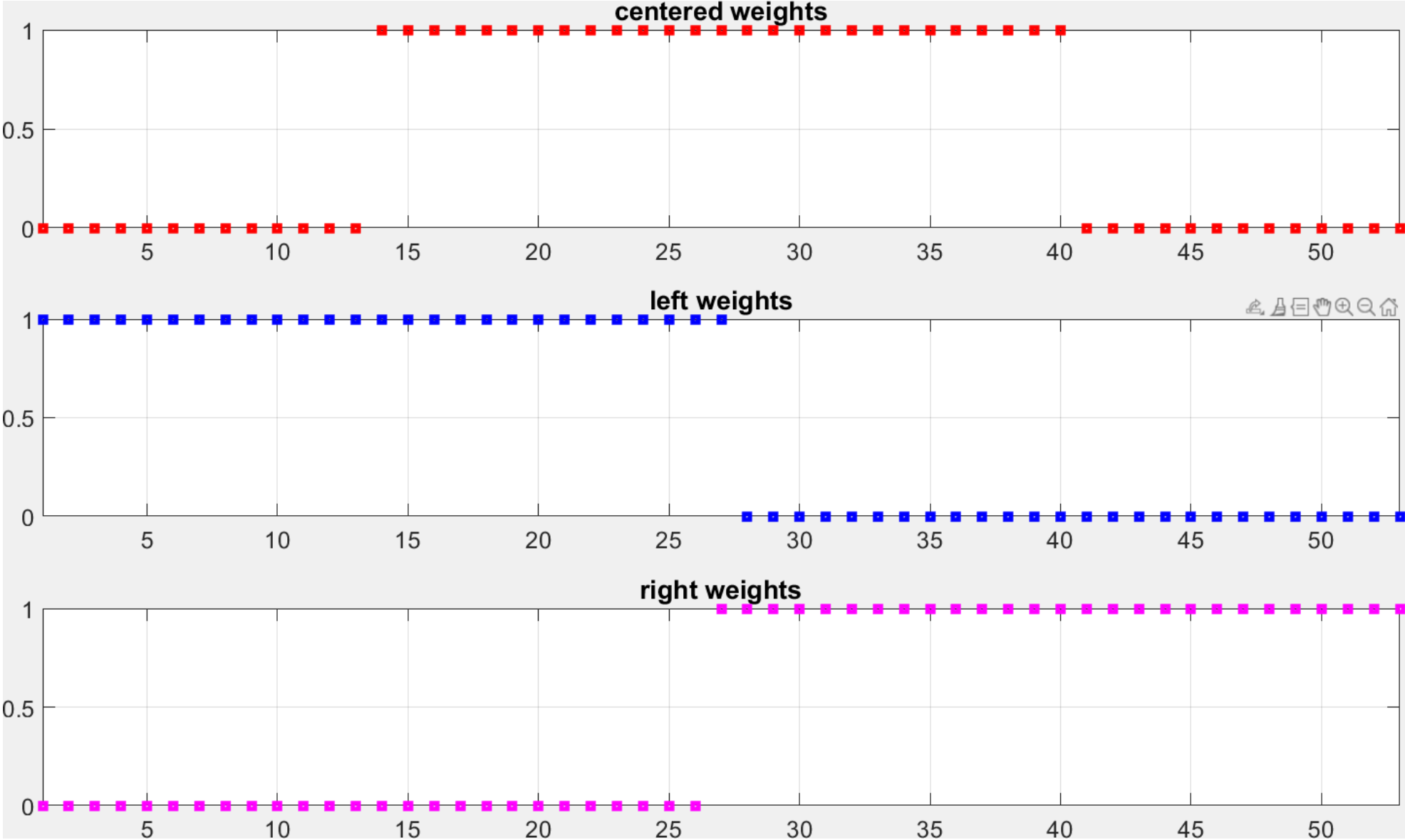
Assignment 2: Weighted LPA Kernels

Lez21_B_weighted_LPA

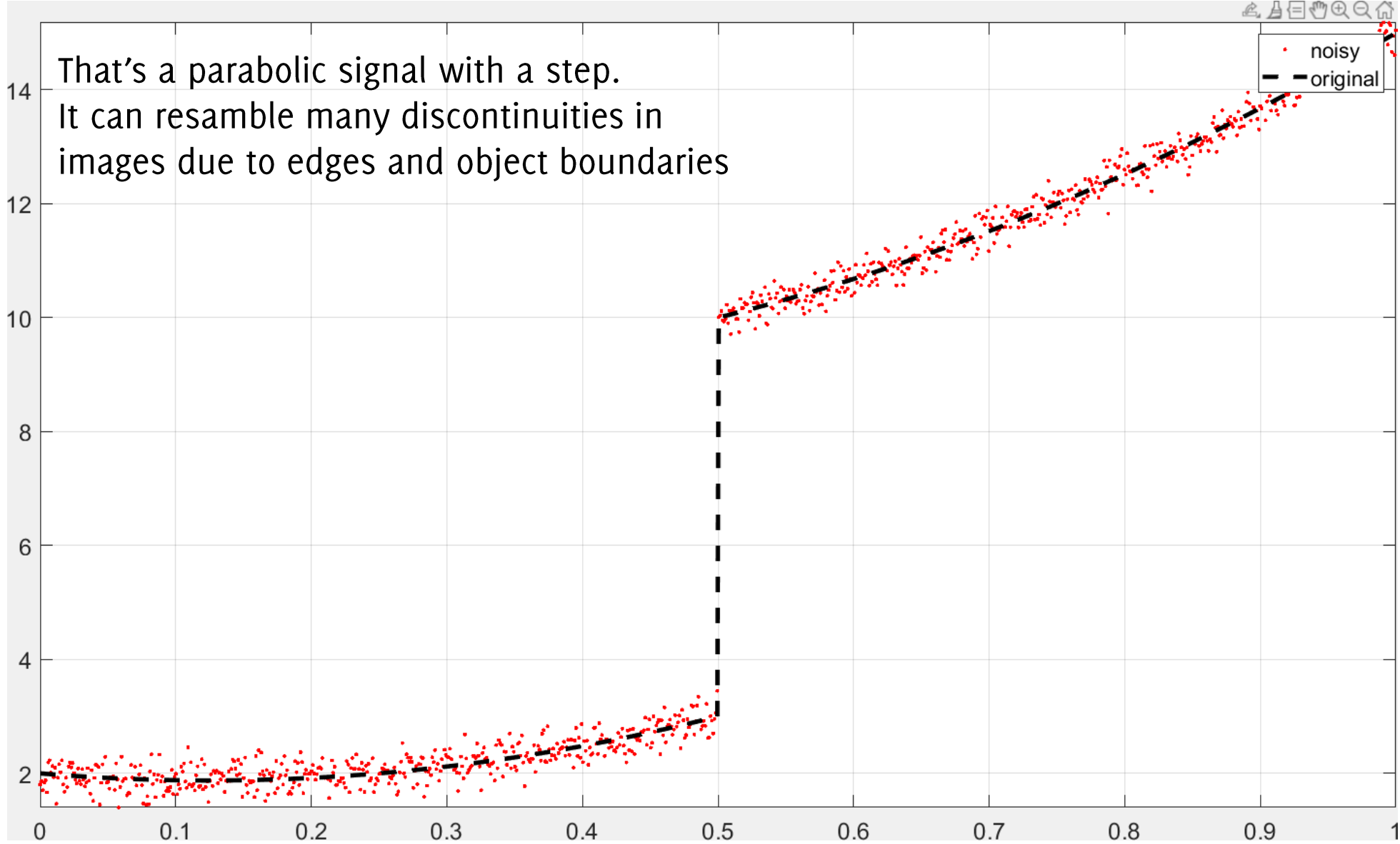
Define the **weighted LPA** filters for a given polynomial order N and over a fixed support M to perform regression over noisy signals

Use binary weights to compute centered, left and right estimates. See how these behave w.r.t. signal discontinuities

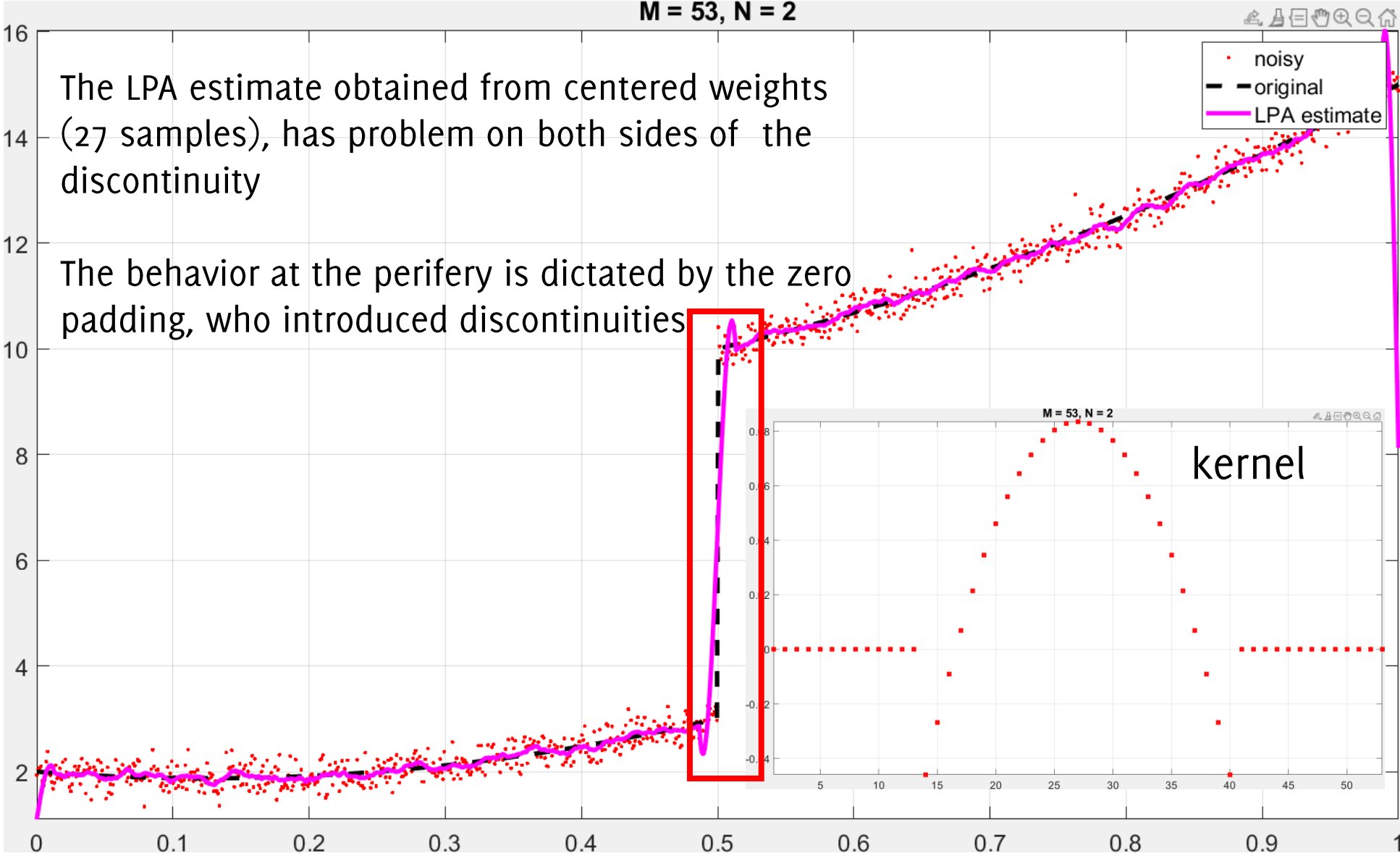
Example of binary weights to use



Handling Discontinuities



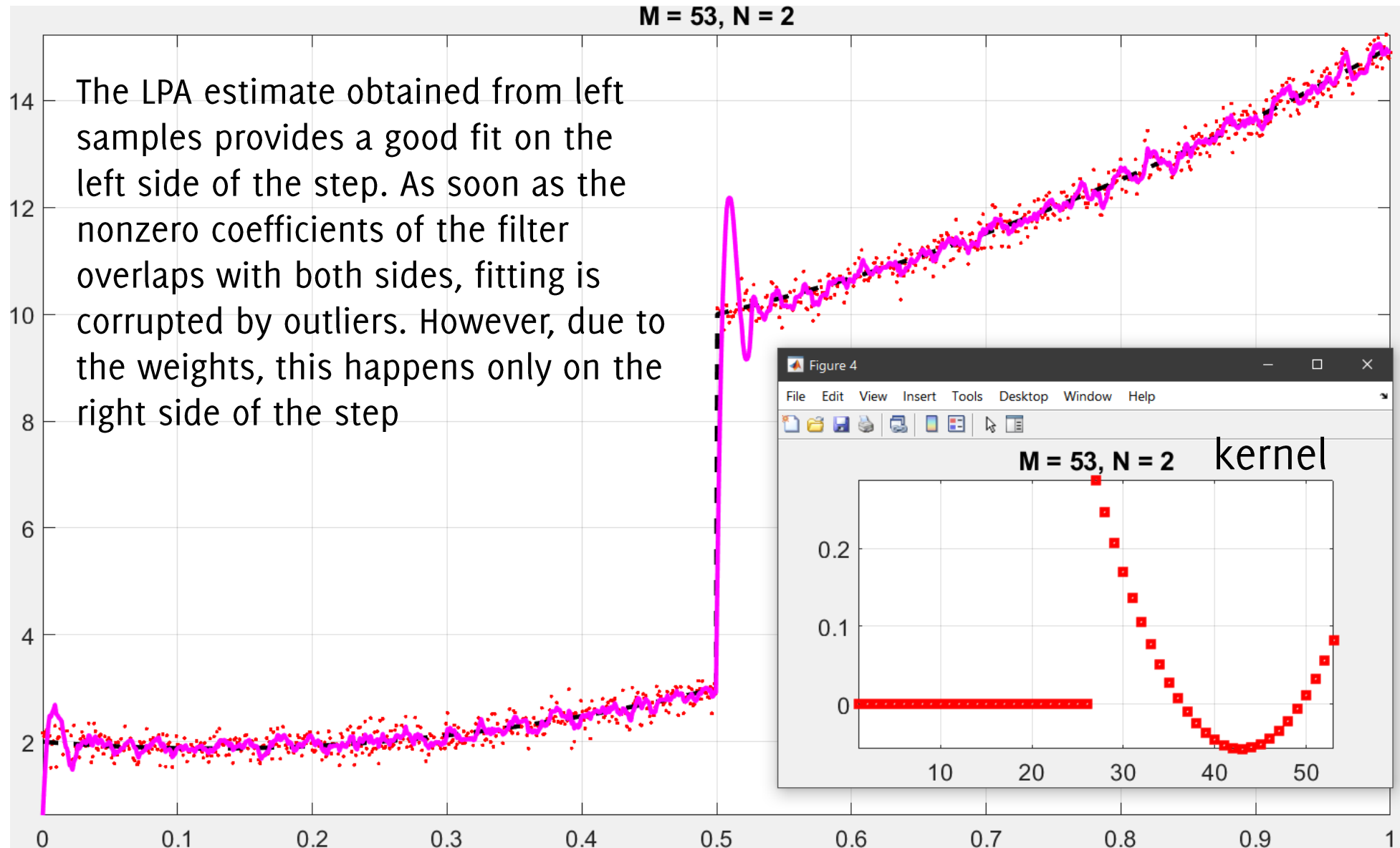
Handling Discontinuities: «centered weights»



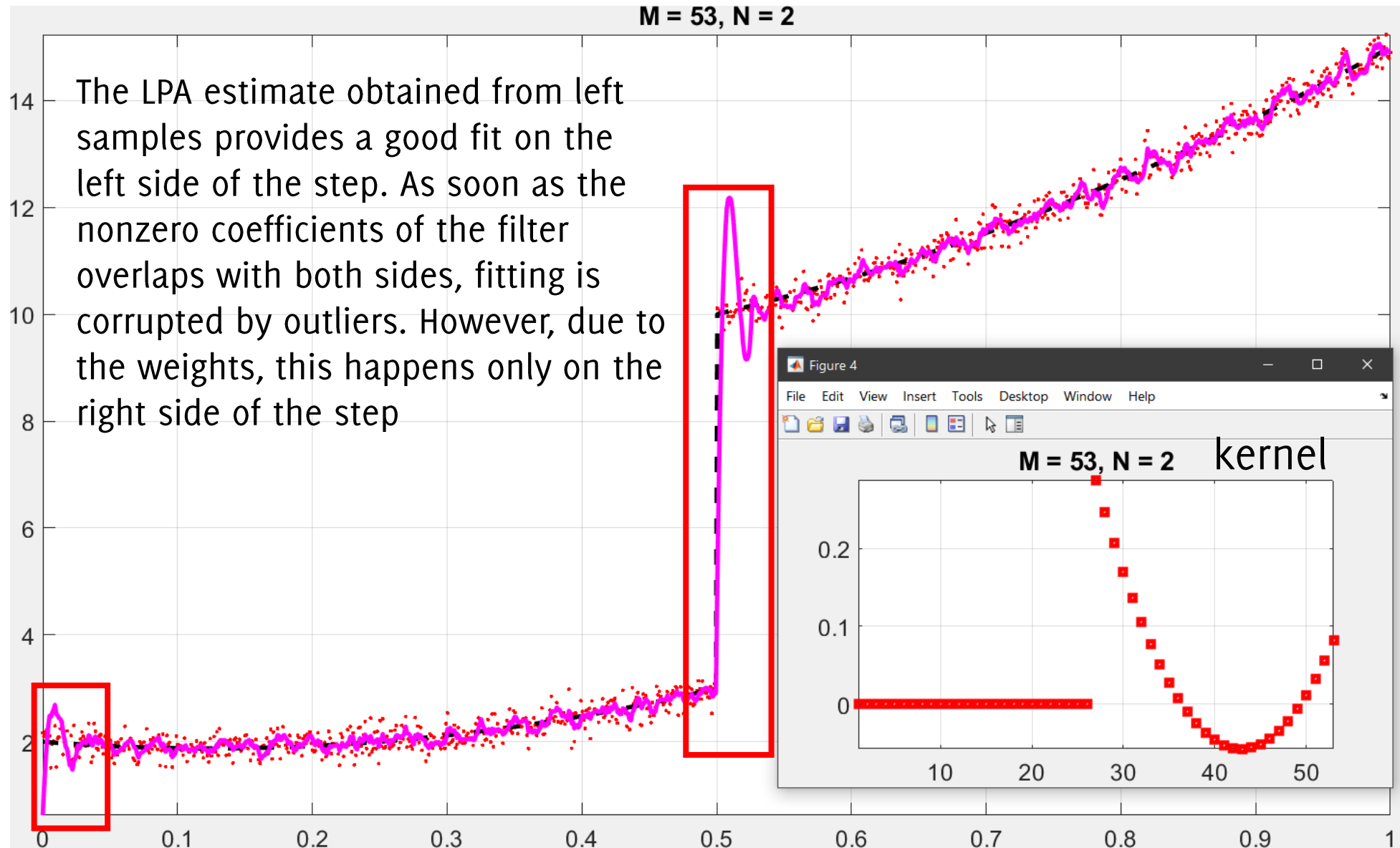
The LPA estimate obtained from centered weights (27 samples), has problem on both sides of the discontinuity

The behavior at the periphery is dictated by the zero padding, who introduced discontinuities

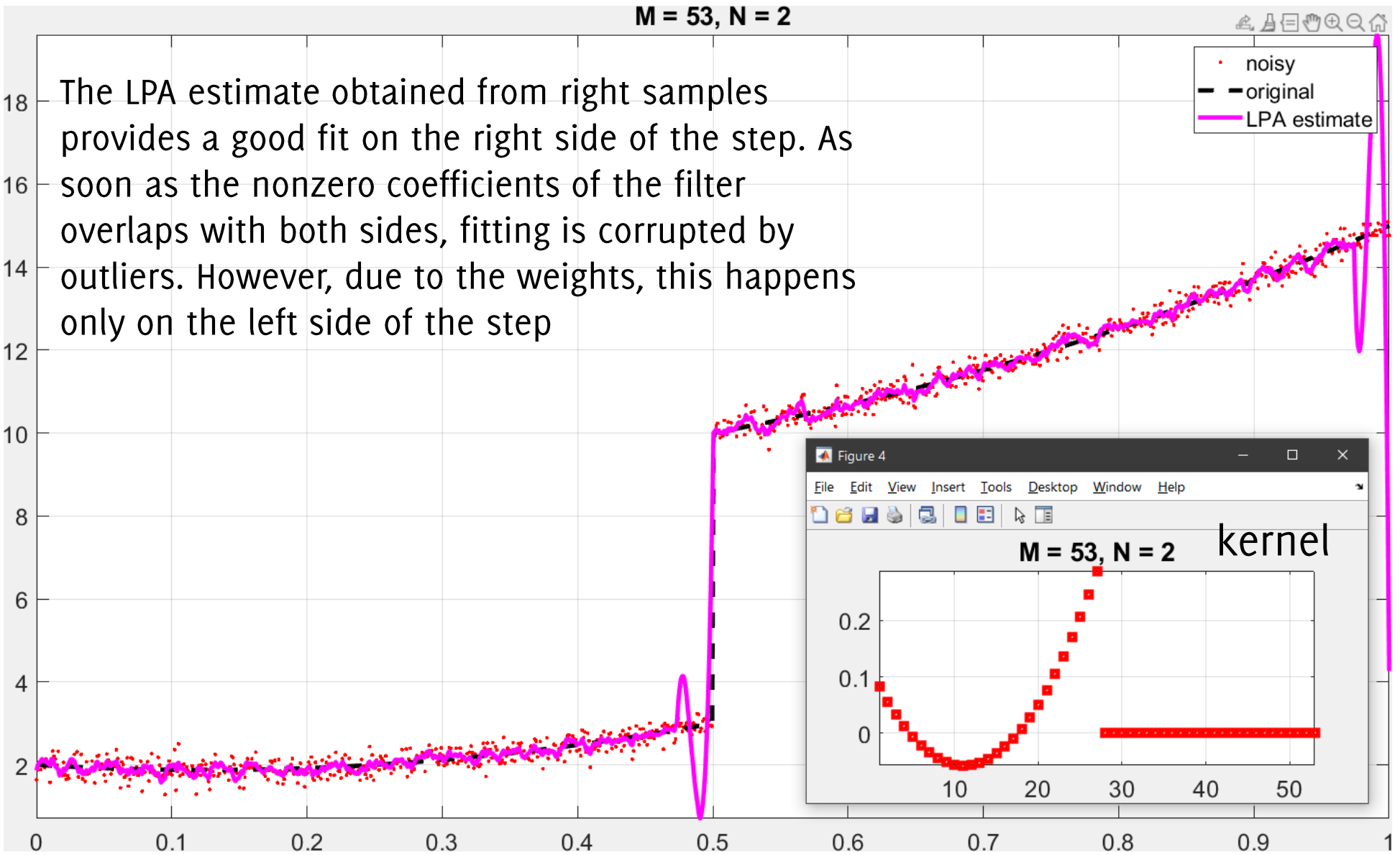
Handling Discontinuities: «left weights»



Handling Discontinuities: «left weights»

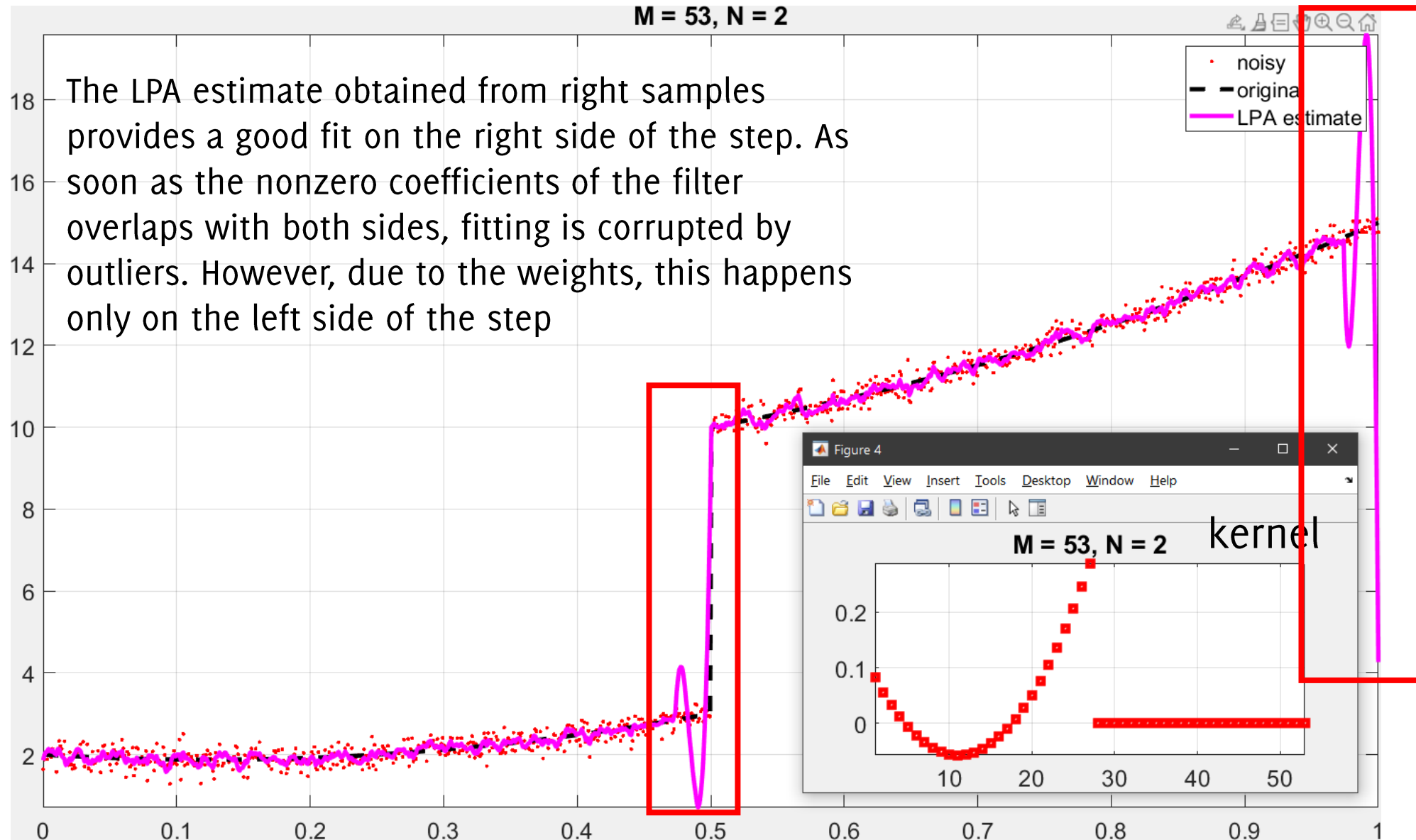


Handling Discontinuities: «right weights»



The LPA estimate obtained from right samples provides a good fit on the right side of the step. As soon as the nonzero coefficients of the filter overlaps with both sides, fitting is corrupted by outliers. However, due to the weights, this happens only on the left side of the step

Handling Discontinuities: «right weights»



Handling Discontinuities: «right weights»

