Patch Modeling and JPEG

Mathematical Models and Methods for Image Processing

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https://boracchi.faculty.polimi.it/teaching/MMMIP.htm

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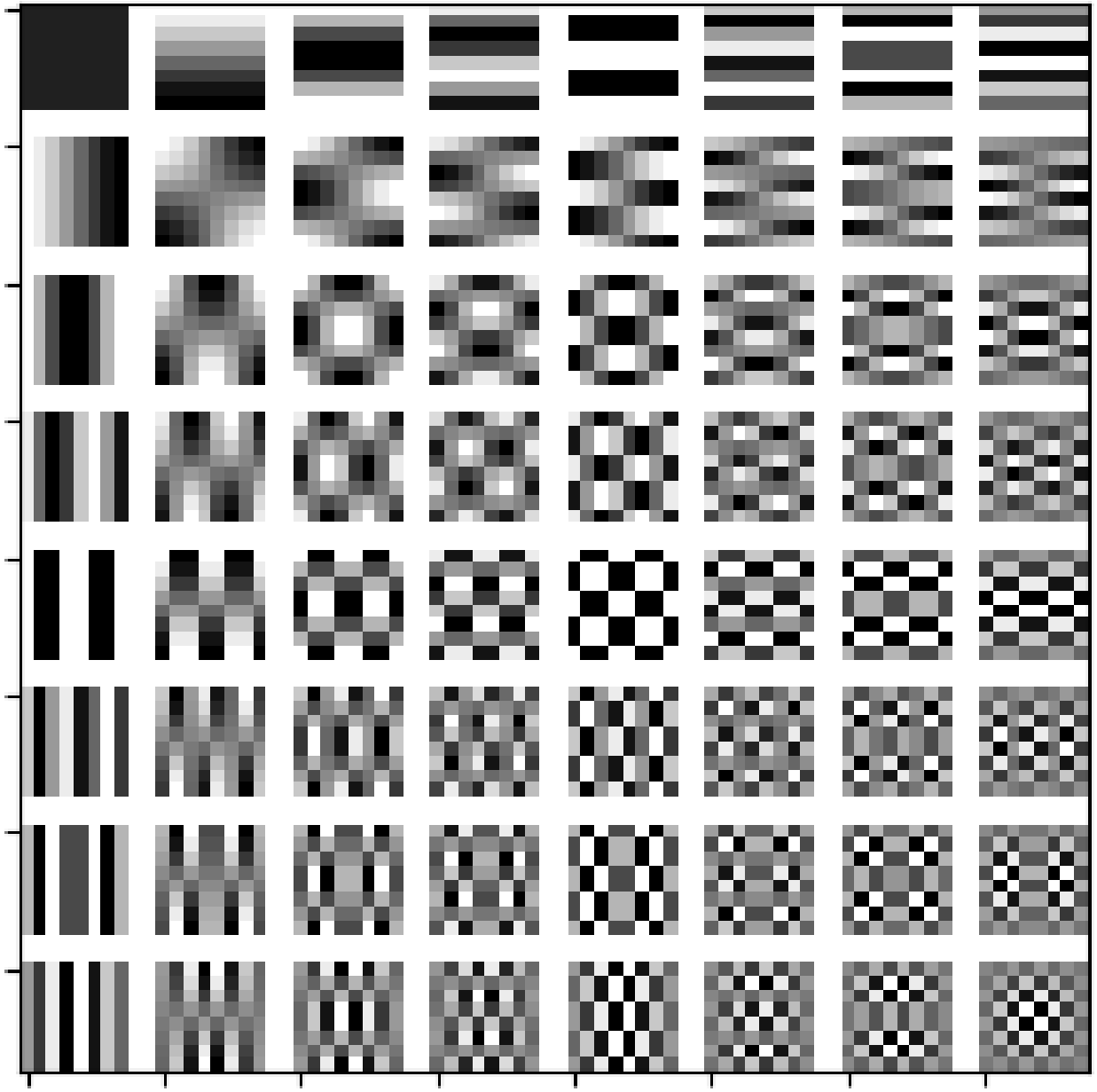
2D DCT

How the atoms in the 2D DCT dictionary look like



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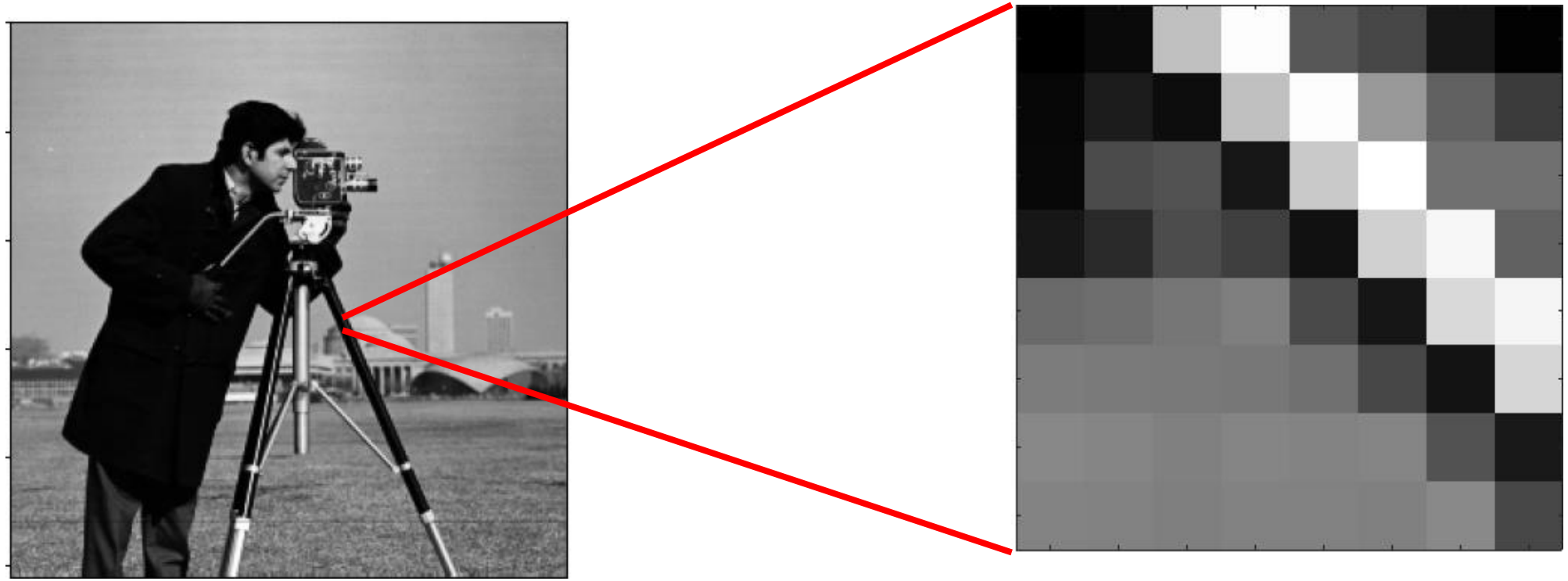
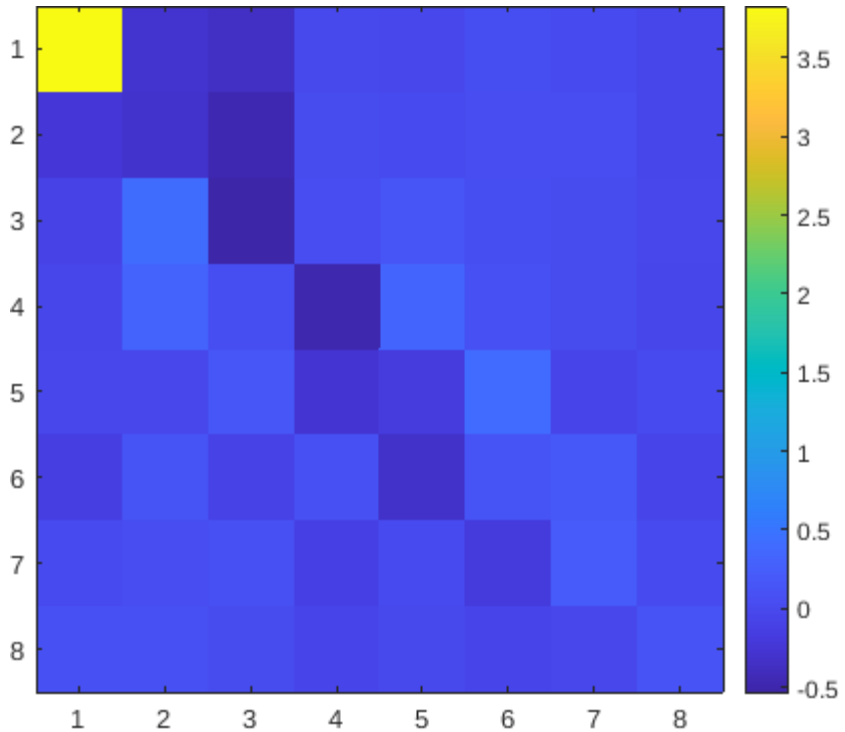
The JPEG Compression

Let’s extract a 8x8 patch from an image



2D DCT of the patch



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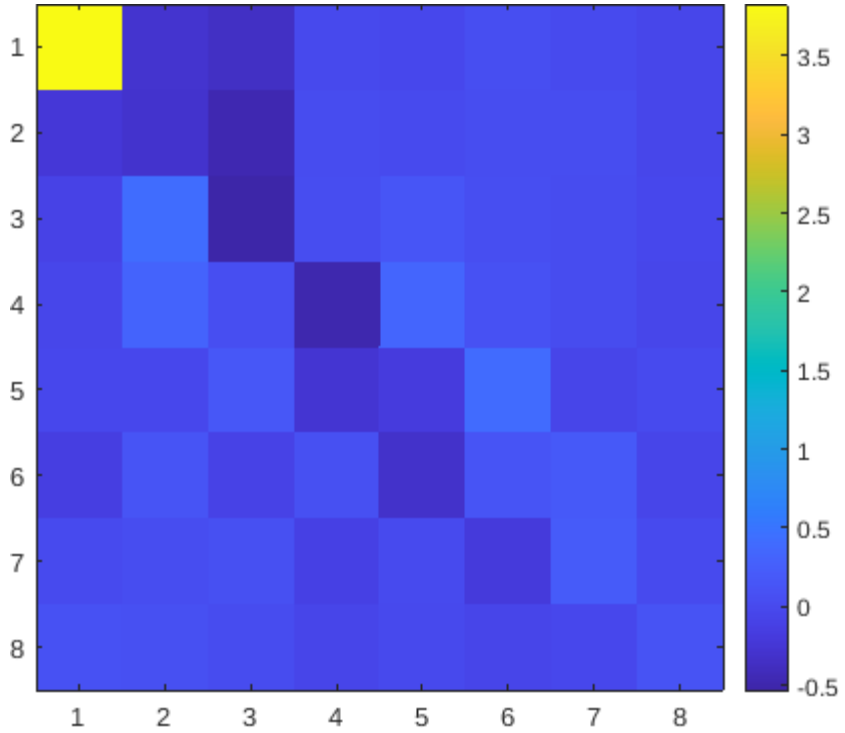
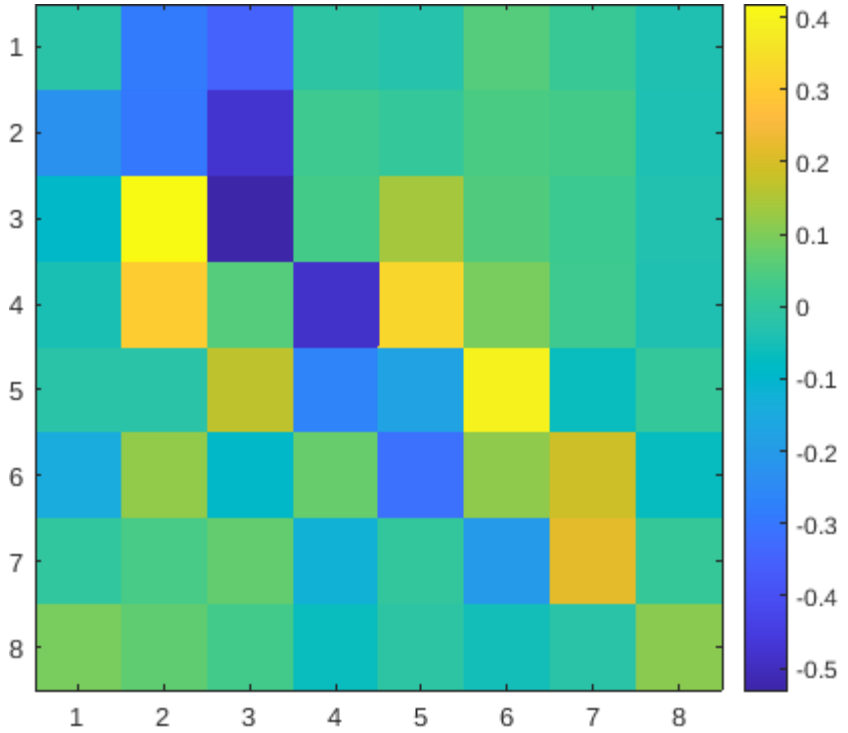
2D DCT of the patch

With DC



Without DC



A lot of coefficients are closed to o!

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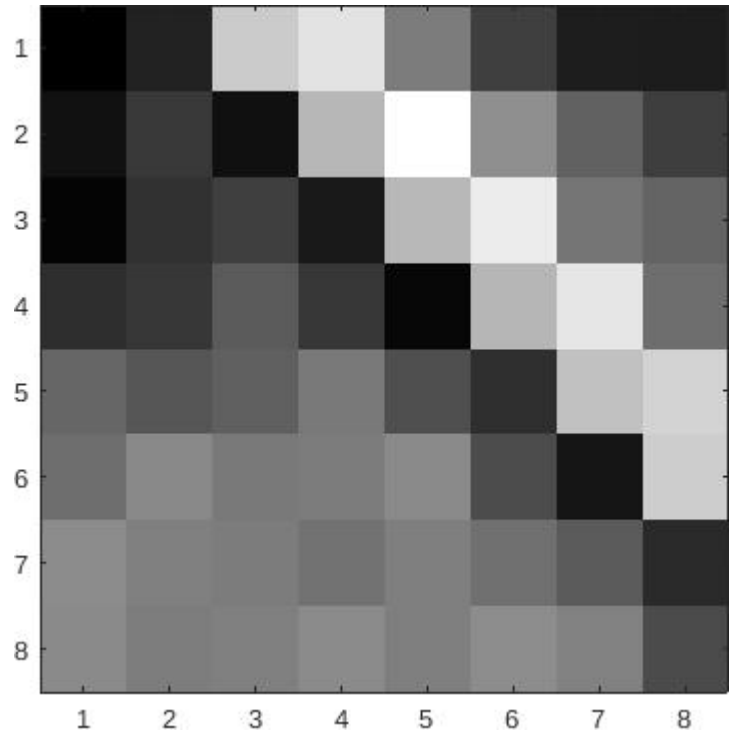
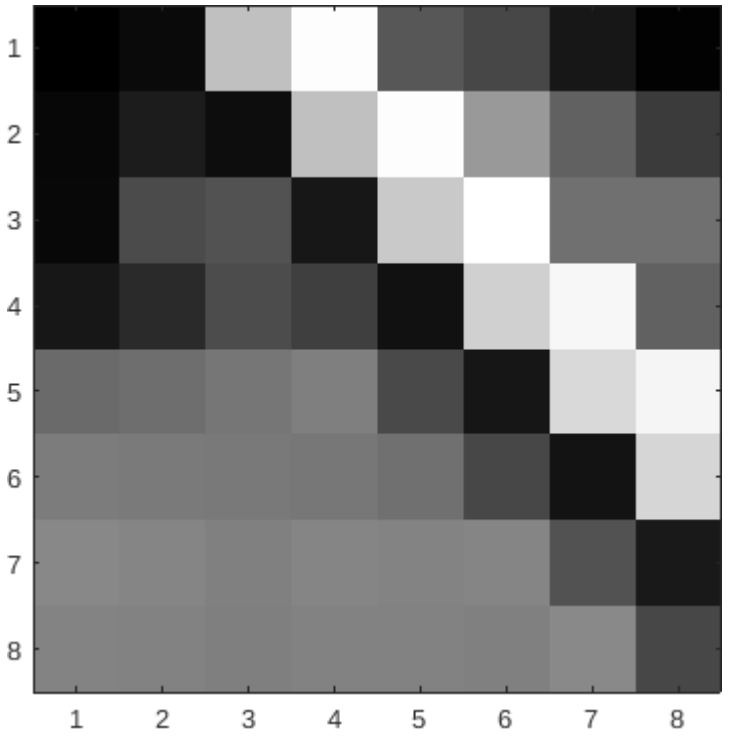
Reconstructed patch

Original Patch

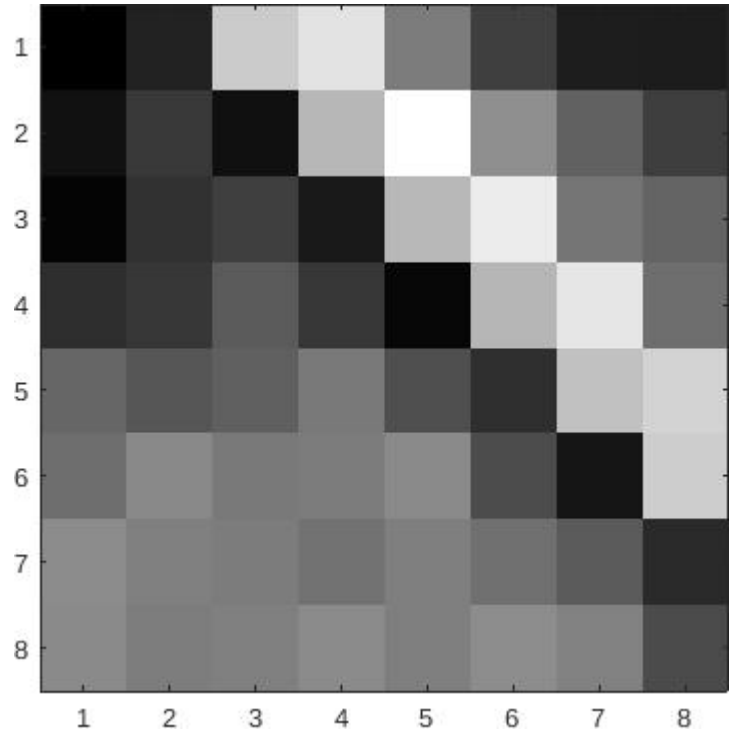


Reconstructed Patch

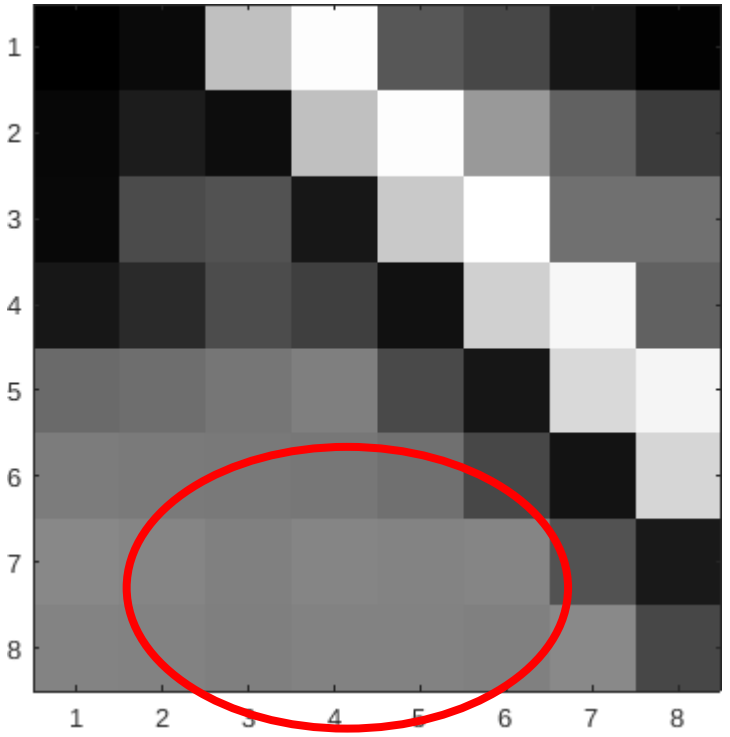


Reconstructed patch

Original Patch



Reconstructed Patch



Smooting in this area

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Assignments

Last Time Assignment: Generate the Basis

- Generate the DCT basis according to the following formula (DCT type II) the k-th atom of the DCT basis in dimension M is defined as

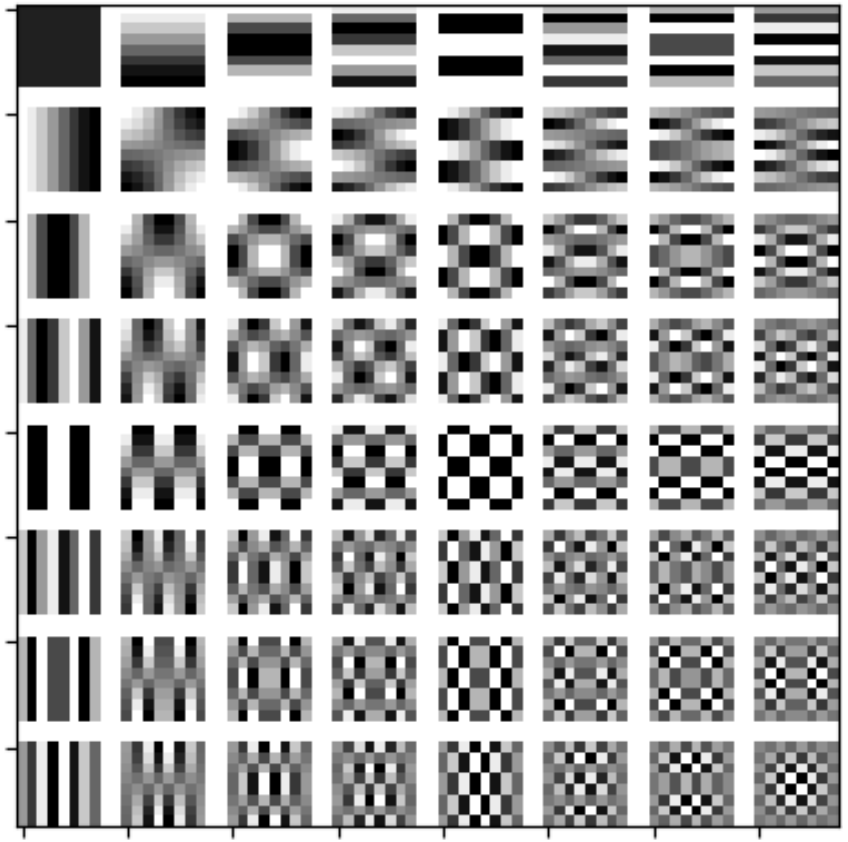
$$DCT_k(n) = c_k \cos\left(k\pi \frac{2n+1}{2M}\right) \; n, k = 0, \ldots, M-1$$

  where $c_0 = \sqrt{1/M}$ and $c_k = \sqrt{2/M}$ for $k \neq 0$.

- How can you use the function `dct` and its inverse `idct` to define the DCT matrix?

First Assignment: 2D DCT dictionary

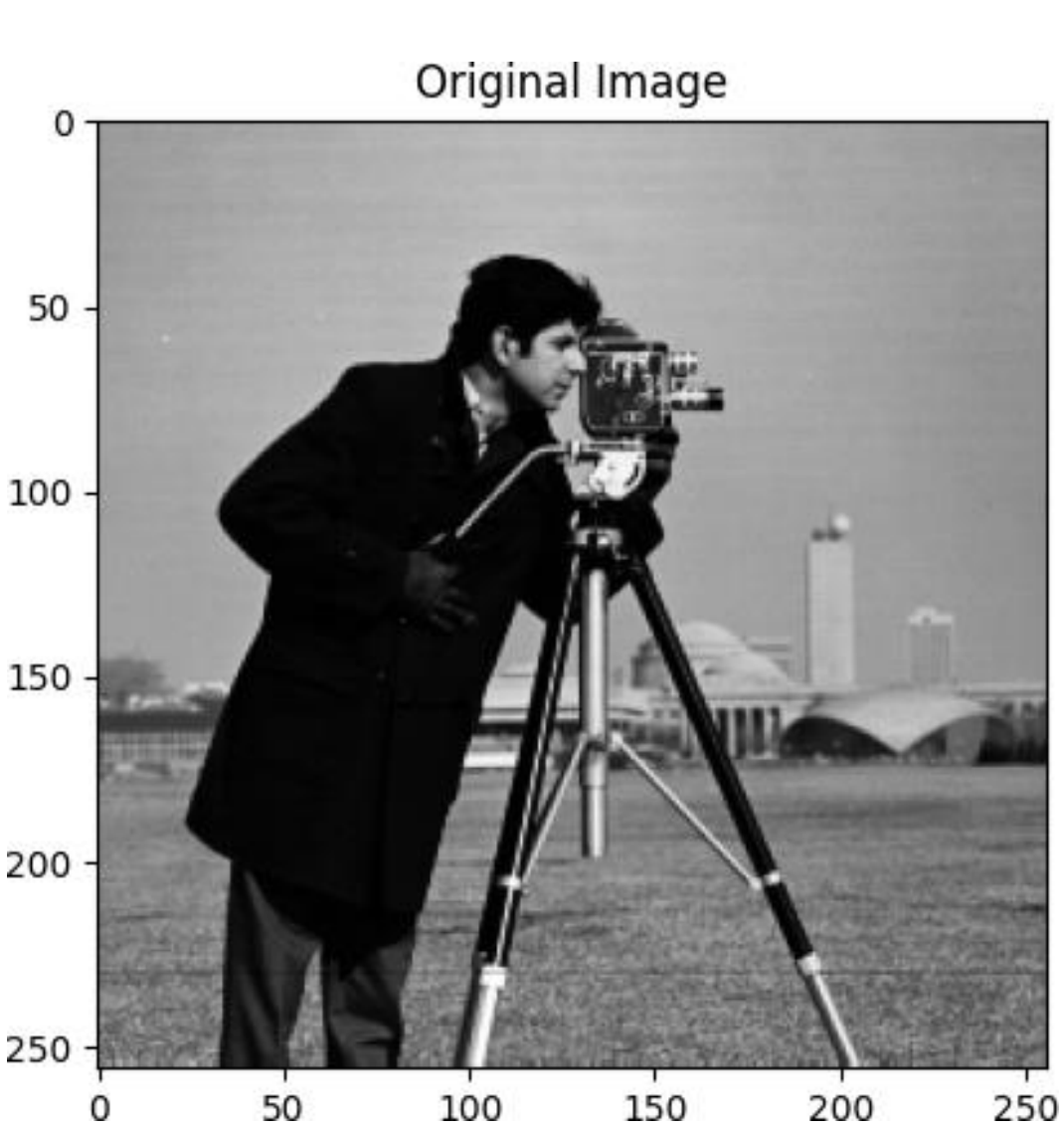
- Generate the 2D DCT dictionary using the `dct2` and `idct2` functions
  - Use this dictionary to compute the representation of a patch
- Generate the 1D DCT dictionary using the `dct` and `idct` functions
  - Use this dictionary to compute the separable 2D DCT of the same patch
- Verify that the coefficients of the two representations are the same

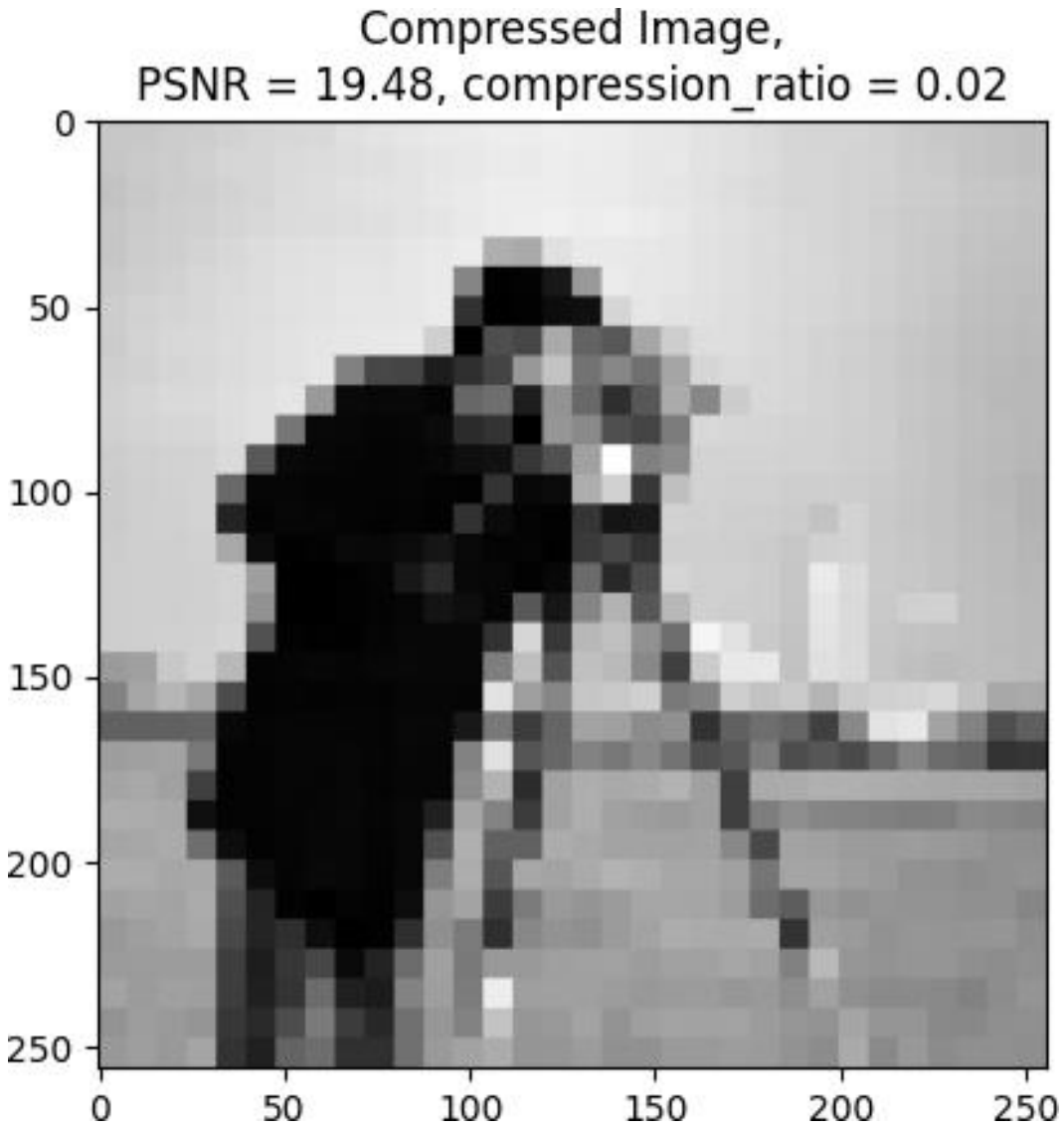


Second Assignment: JPEG Compression

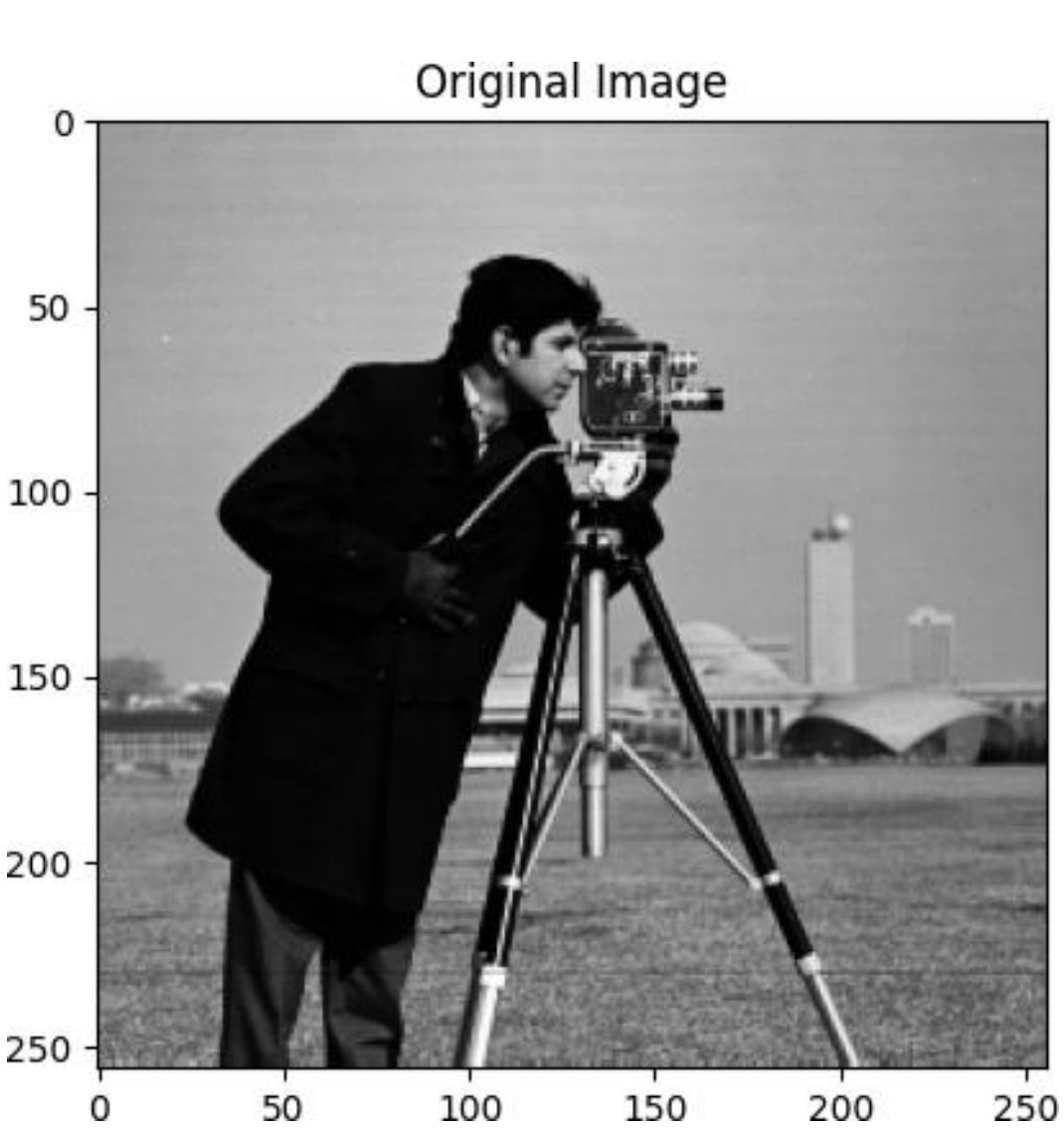
- Implement the JPEG compression algorithm
- Compute the PSNR of the compressed image
- Compute the compression ratio
- Try different thresholds: how the thresholds affect the compression?
- Run the JPEG compression on the test image and try different threshold. What can you observe?

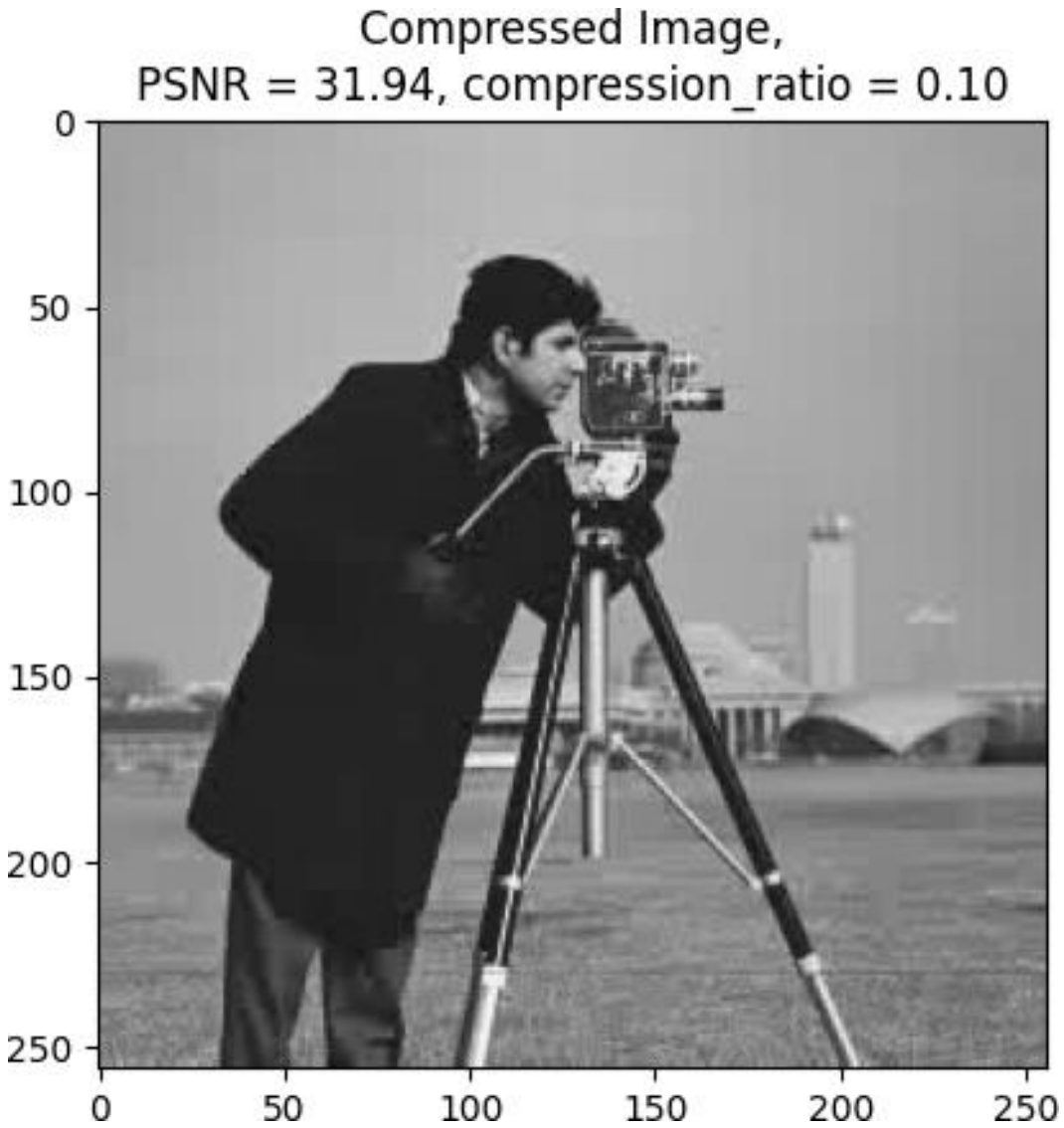
Second Assignment: JPEG Compression





Second Assignment: JPEG Compression





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