Patch Modeling and JPEG

Mathematical Models and Methods for Image Processing

Diego Carrera

https://boracchi.faculty.polimi.it/teaching/MMMIP.htm

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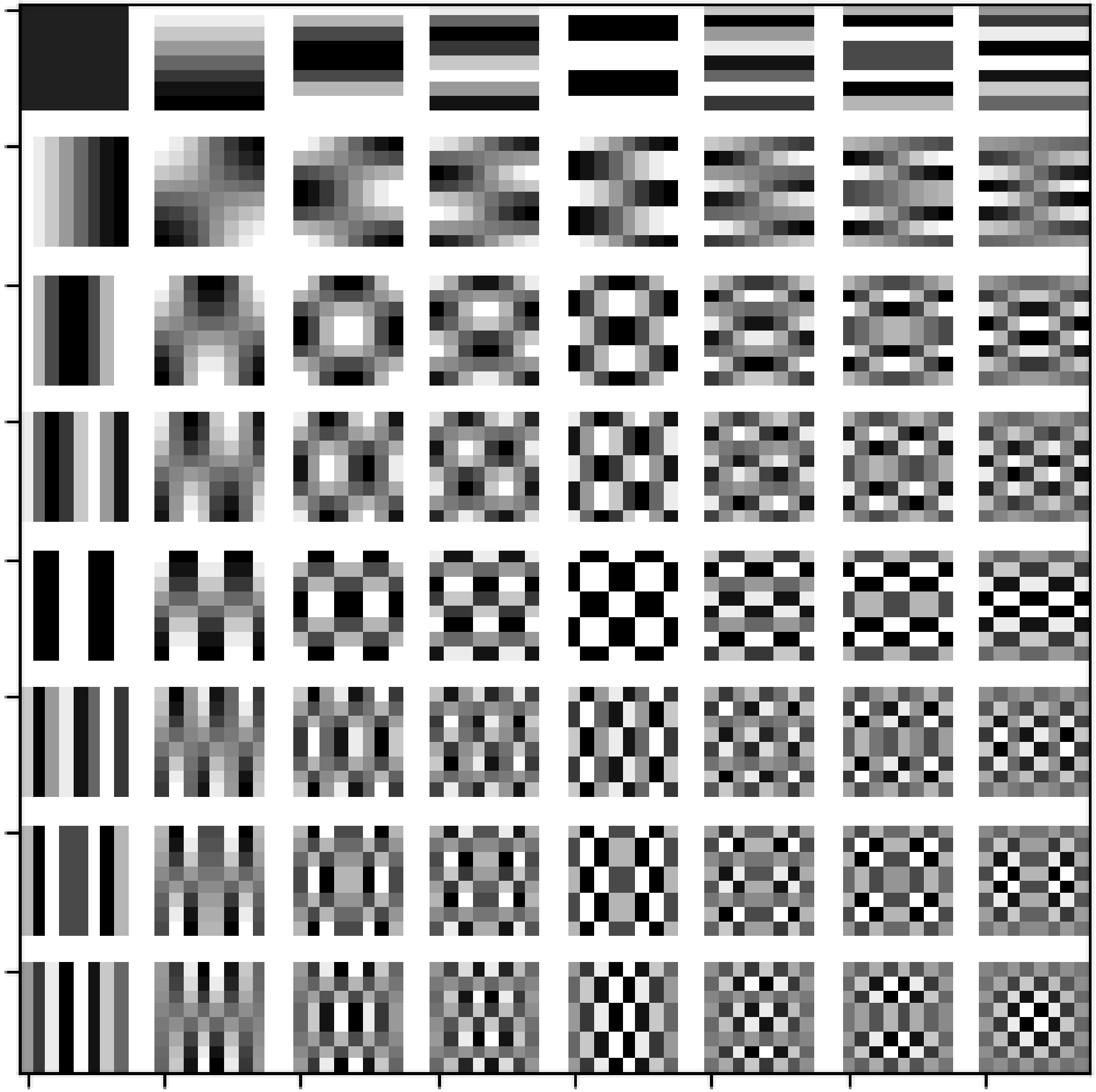
2D DCT

How the atoms in the 2D DCT dictionary look like



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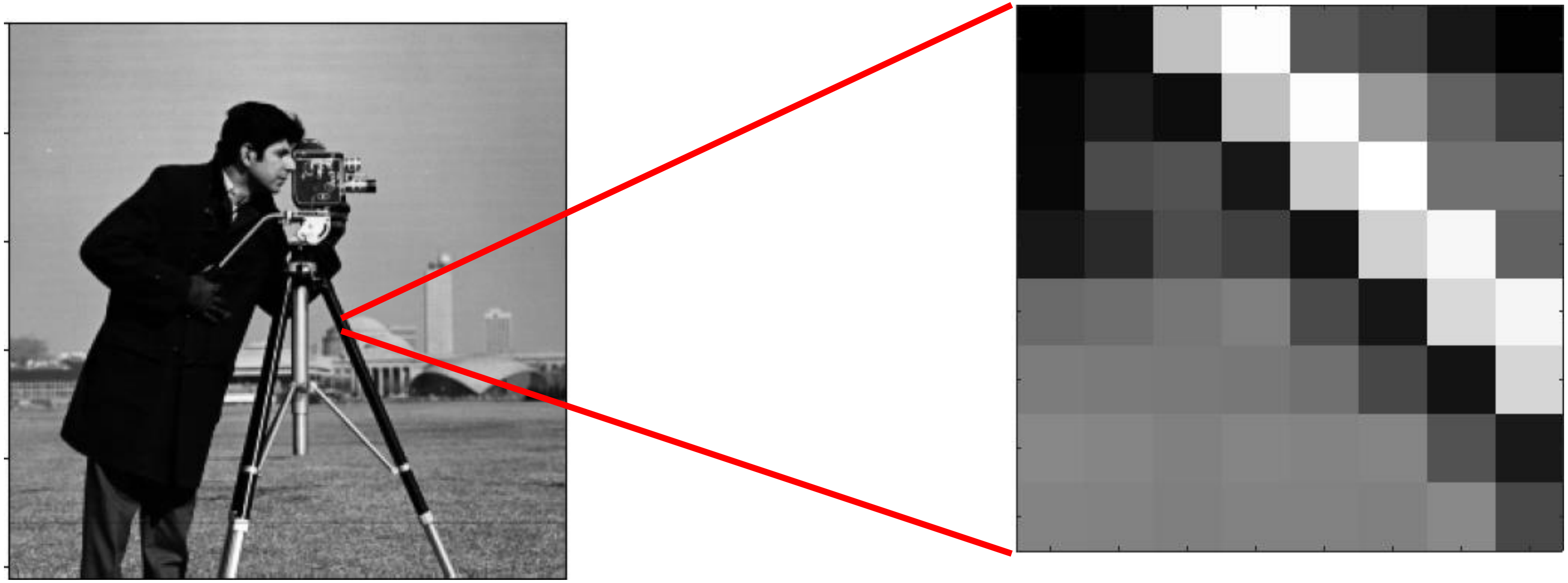
The JPEG Compression

Let’s extract a 8x8 patch from an image



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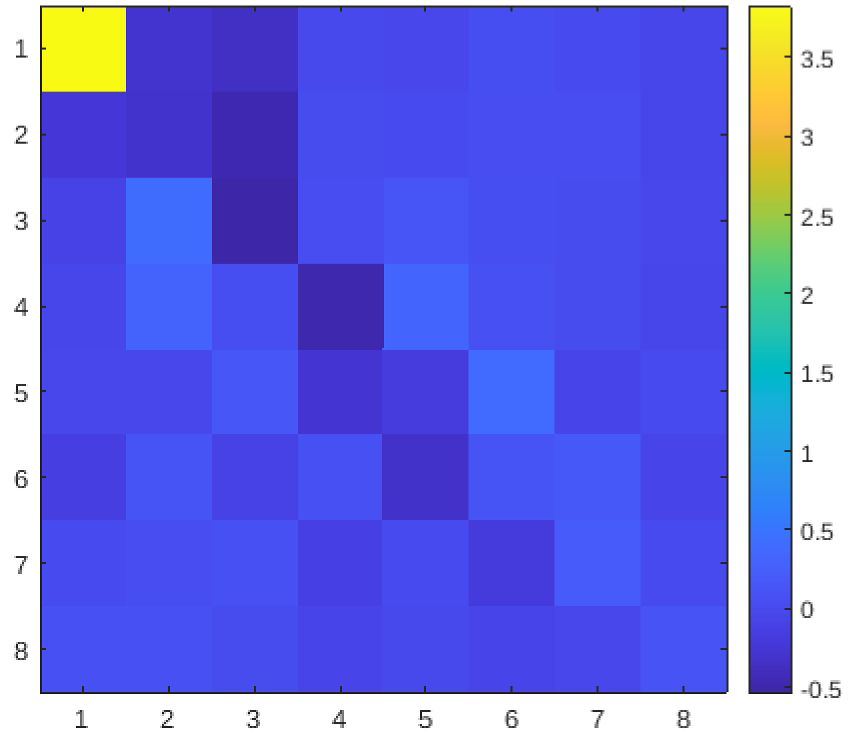
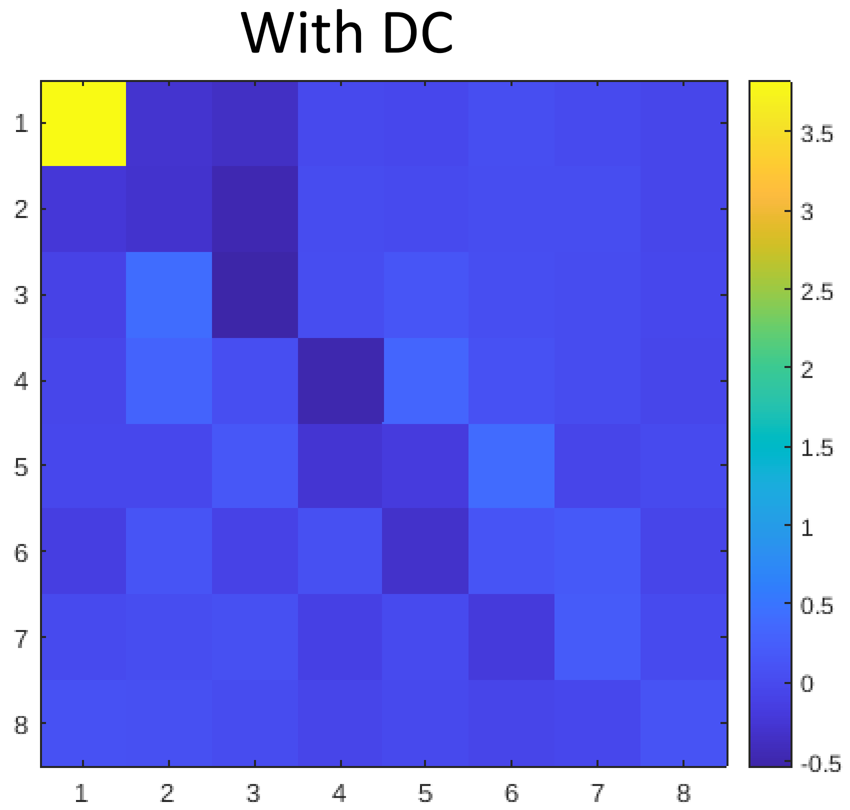
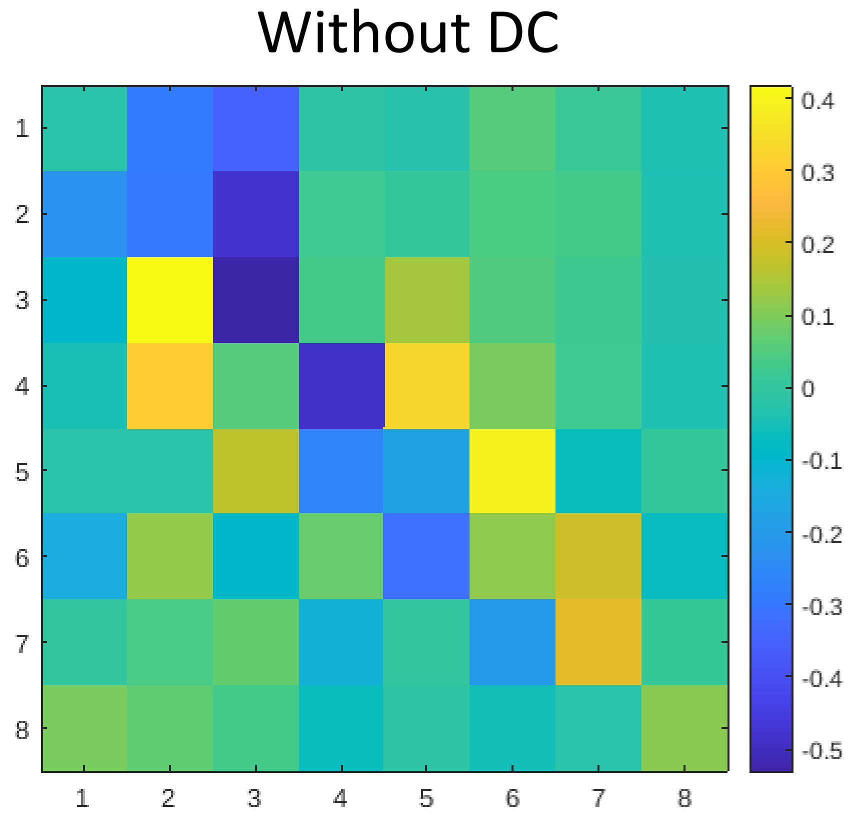
2D DCT of the patch



2D DCT of the patch





A lot of coefficients are closed to 0!

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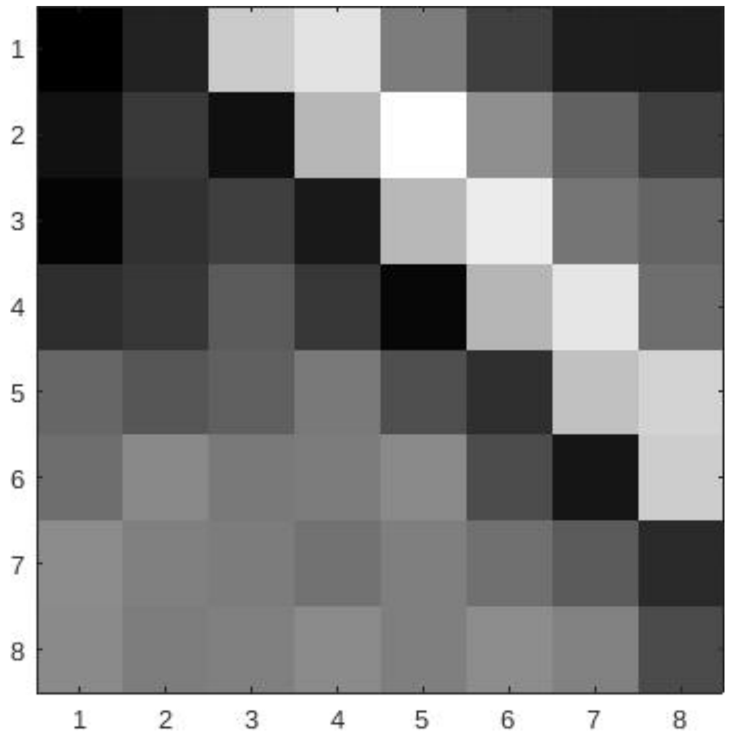
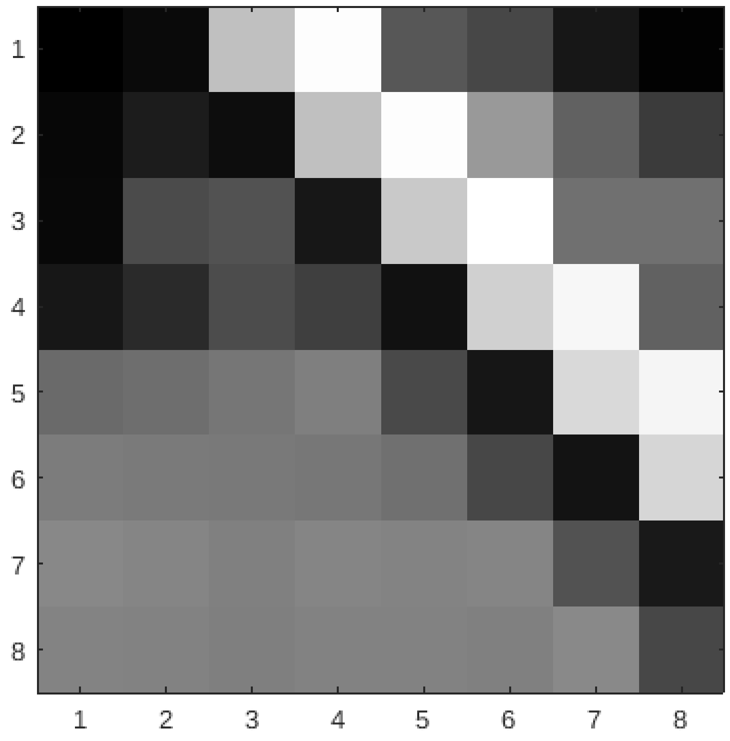
Reconstructed patch

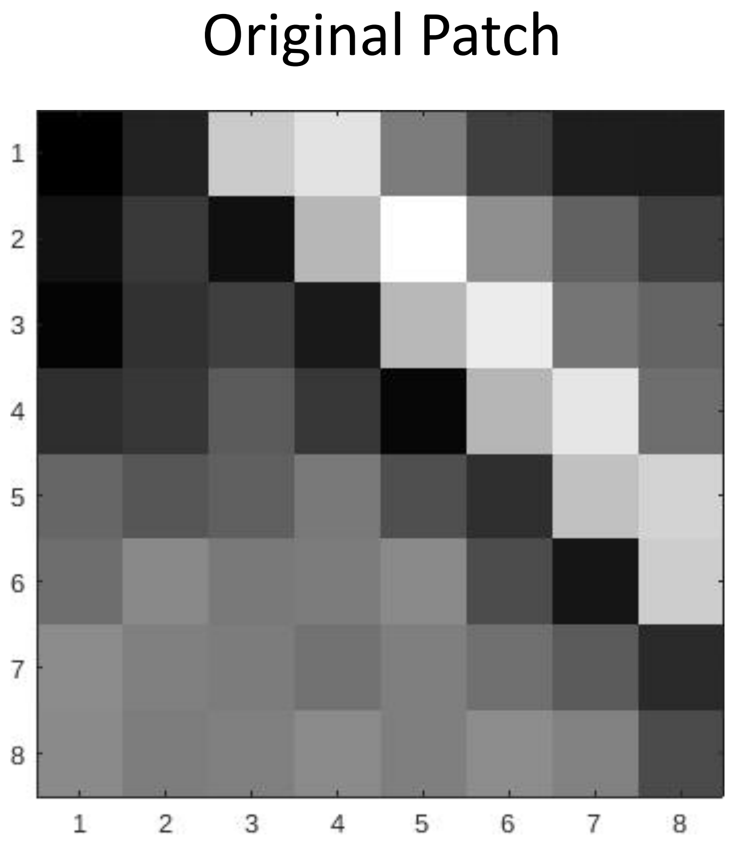
Original Patch

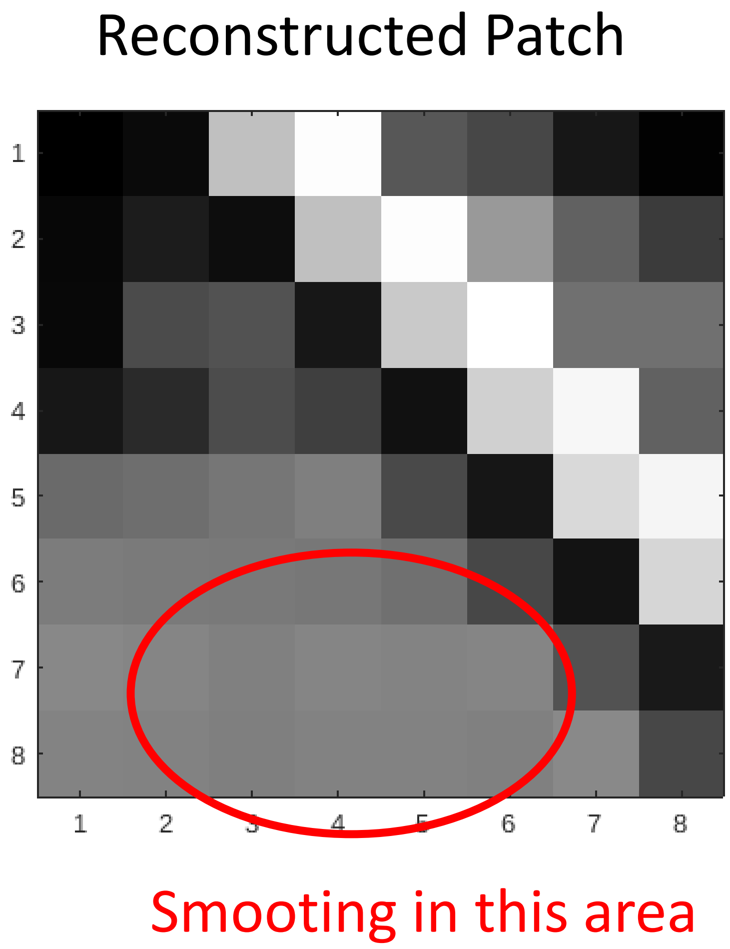


Reconstructed Patch



Reconstructed patch





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Denoising

Denoising: The Issue

A Detail in Camera Raw Image S

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Denoising: The Issue

Denoised $\hat{Y}$

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Denoising: The Issue

A Detail in Camera Raw Image S

Denoising: The Issue

Denoised $\hat{Y}$

Image Formation Model

Observation model is

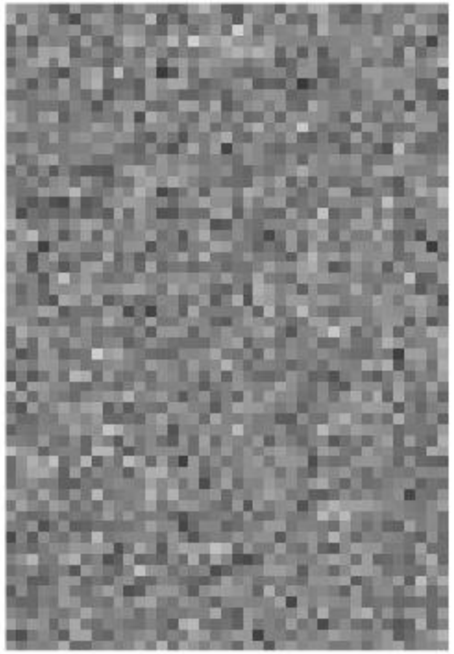
$$z(x) = y(x) + \eta(x), \qquad x \in \mathcal{X}$$

z

$=$

y

$+$



η

Image Formation Model

Observation model is

$$S(i) = Y(i) + \eta(i), \qquad i \in \mathcal{X}$$

Where

- i denotes the pixel coordinates in the domain $\mathcal{X} \subset \mathbb{Z}^2$
- Y is the original (noise-free and unknown) image, $y \in [0,1]$
- S is the noisy observation, $S \in [0,1]$ (clipping)
- η is the noise realization

For the sake of simplicity we assume Additive White Gaussian Noise (AWGN):

$\eta \sim N(0, \sigma^2)$ and $\eta(x)$ are all independent realizations.

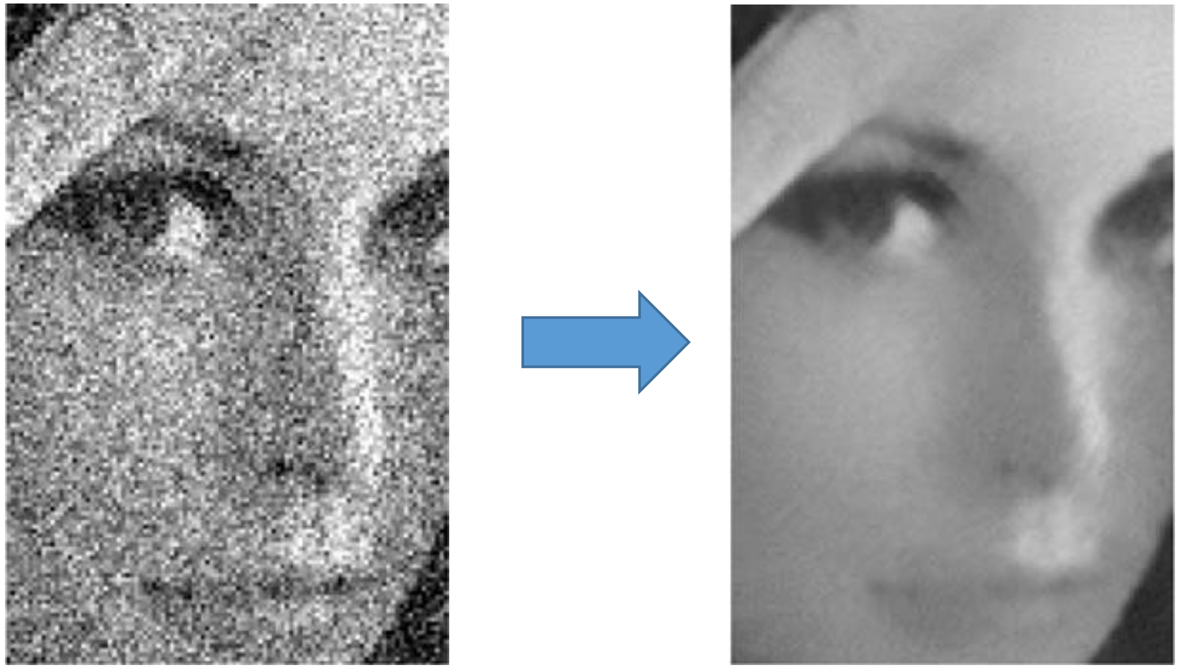
The noise standard deviation σ is also assumed as known.

Goal of Image Denoising

The goal of **image denoising** is to compute $\hat{y}$ *realistic* estimate of the original image y, given the noisy observation z

Denoising is an **ill posed problem** and requires some form of **regularization** to promote outputs that are close to natural images.

Our Prior: **Sparsity w.r.t. DCT basis!**



s $\hat{Y}$

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Image Denoising

Deniosing is a fundamental step in image processing pipelines

- Improves the quality of digital images to the standard we are used to
- Eases the following algorithms in imaging pipelines from those solving low-level (e.g., edge detection), till high-level (recognition) problems
- It is also a tool to quantitativelly assess the performance of a descriptive model for images.

Assignments

Last Time Assignment: Generate the Basis

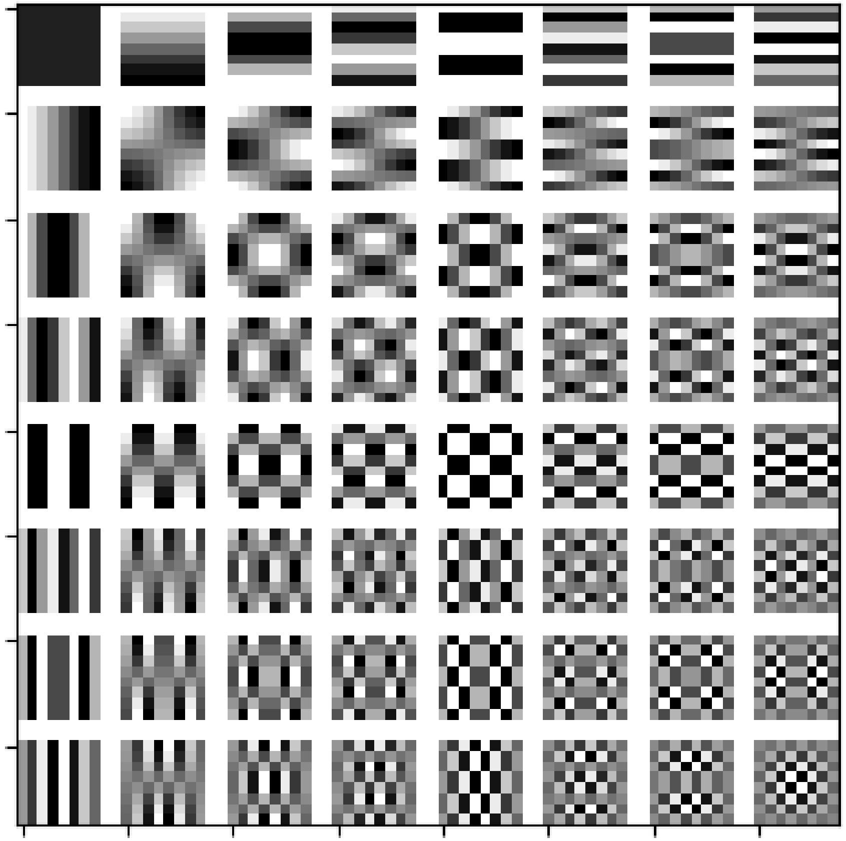
- Generate the DCT basis according to the following formula (DCT type II) the k-th atom of the DCT basis in dimension M is defined as

$$DCT_k(n) = c_k \cos\left(k\pi \frac{2n+1}{2M}\right) \; n, k = 0, \dots, M-1$$

where $c_0 = \sqrt{1/L}$ and $c_k = \sqrt{2/L}$ for $k = 0$.

- How can you use the function dct and it’s inverse idct to define the DCT matrix?

First Assignment: 2D DCT dictionary

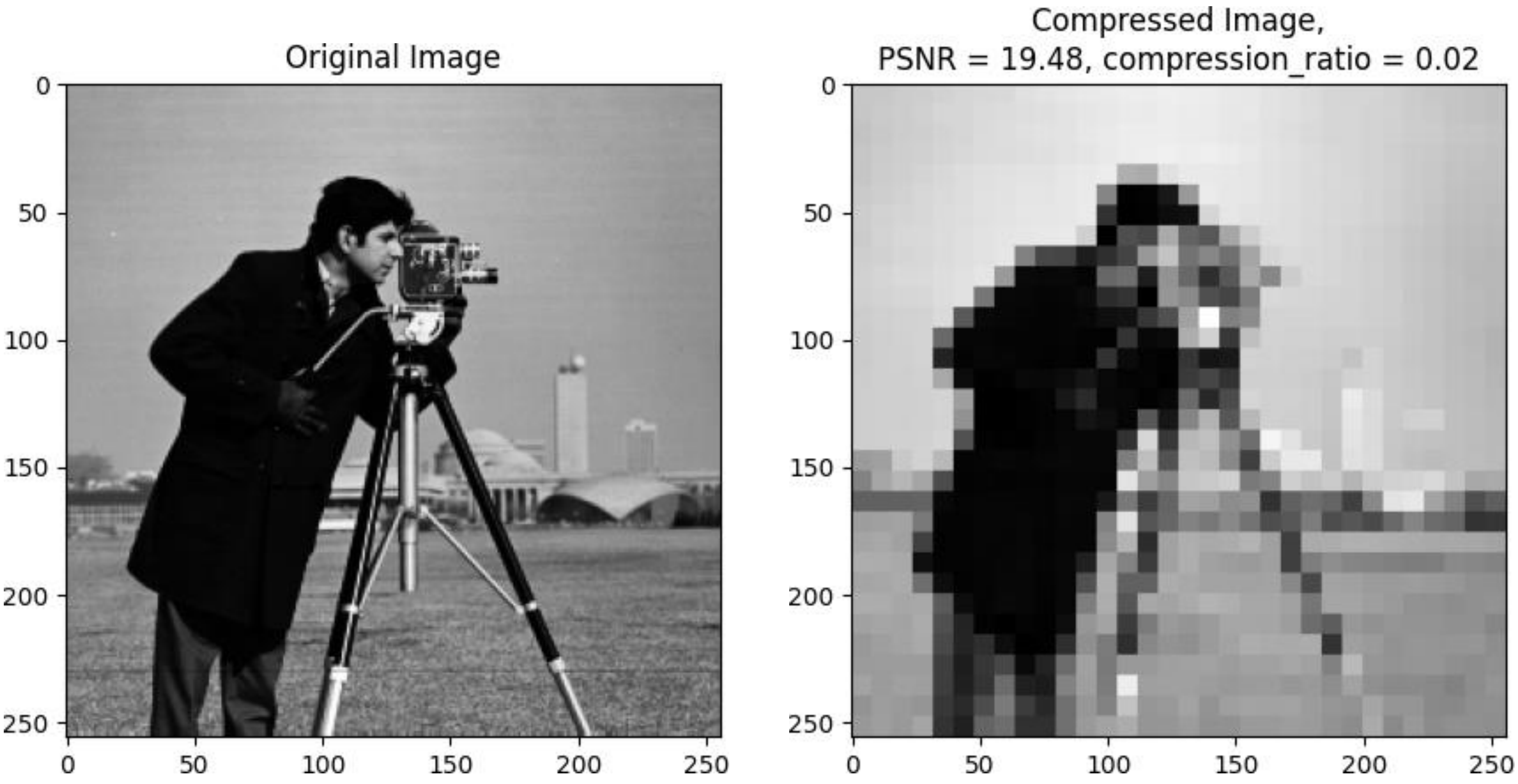


- Generate the 2D DCT dictionary using the dct2 and idct2 function
  - Use this dictionary to compute the representation of a patch
- Generate the 1D DCT dictionary using the dct and idct function
  - Use this dictionary to compute the separable 2D DCT of the same patch
- Verify that the coefficients of the two representations are the same

Second Assignment: JPEG Compression

- Implement the JPEG compression algorithm
- Compute the PSNR of the compressed image
- Compute the compression ratio
- Try different thresholds: how the thresholds affect the compression?

Second Assignment: JPEG Compression



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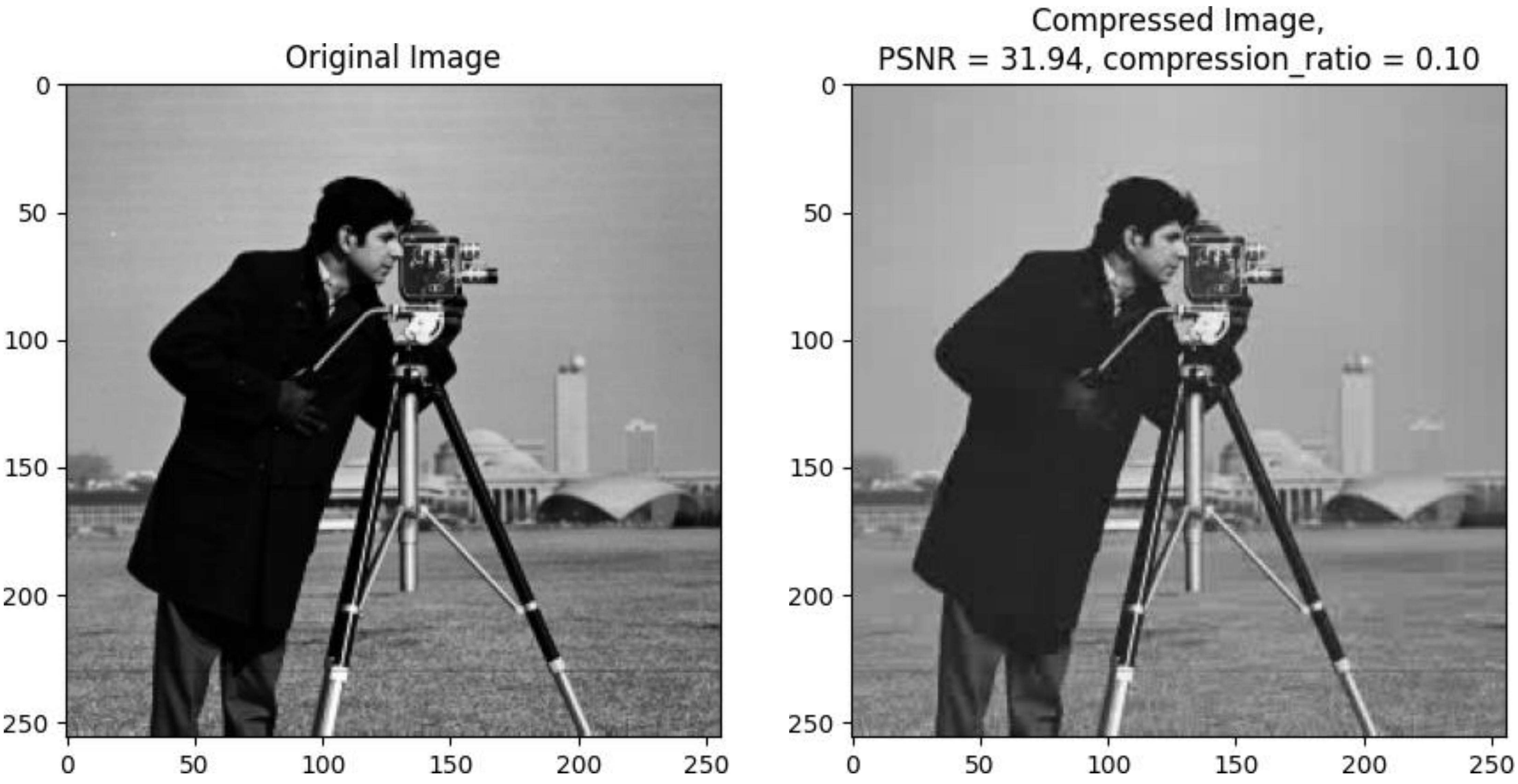
Second Assignment: JPEG Compression



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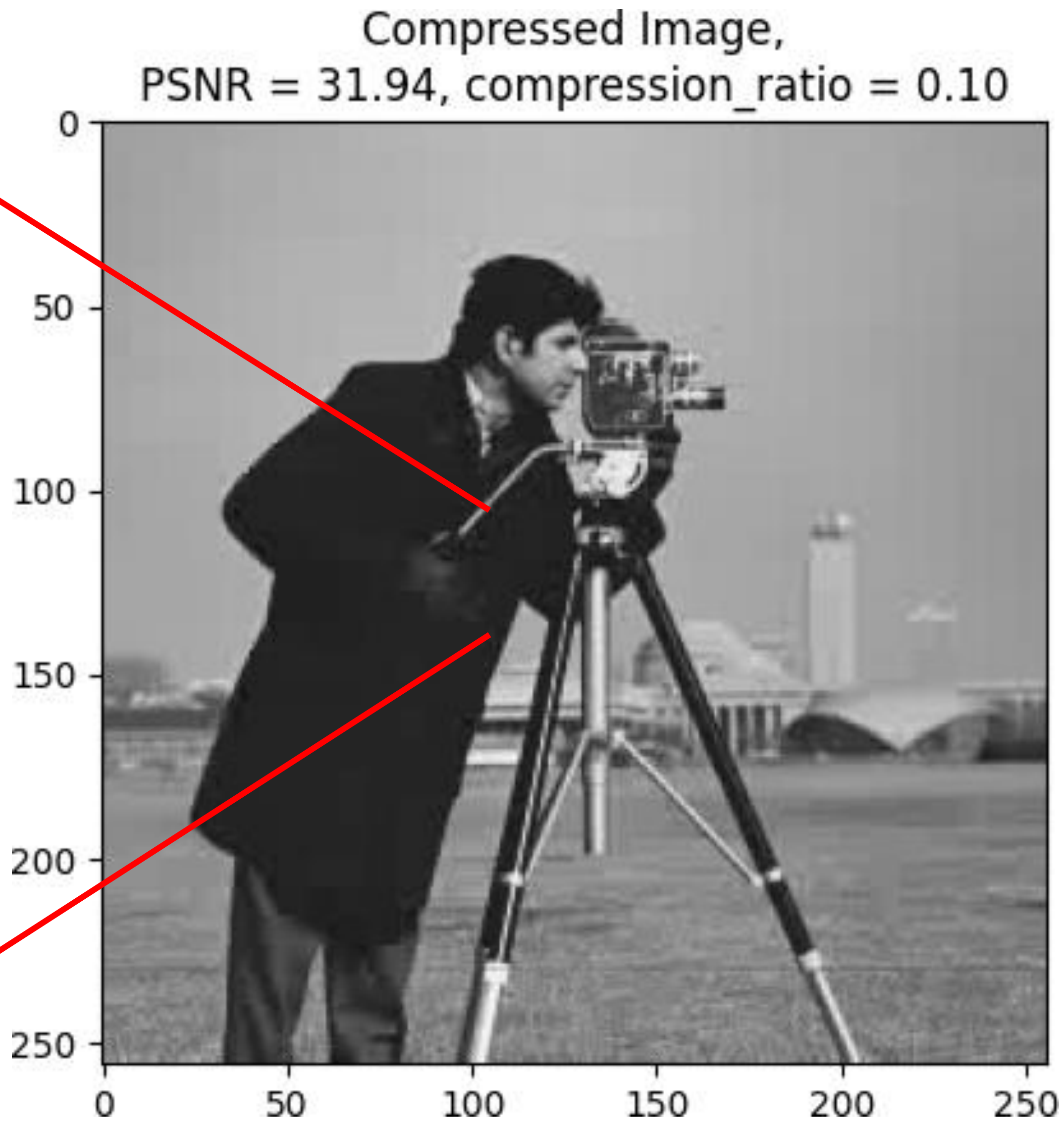
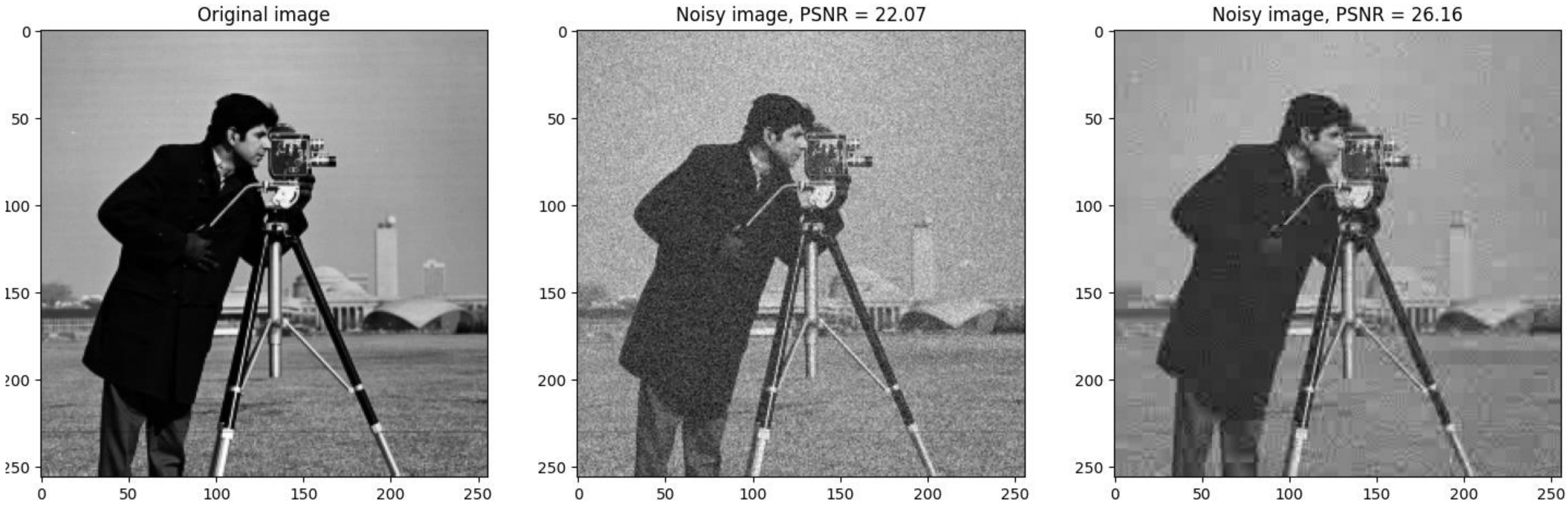
Second Assignment: JPEG Compression



Third Assignment: Denoising

- Implement the DCT fenoising algorithm
- Compute the PSNR of the compressed image


Original image
Noisy image, PSNR = 22.07
Noisy image, PSNR = 26.16
0 50 100 150 200 250
0 50 100 150 200 250
0 50 100 150 200 250
0 50 100 150 200 250
0 50 100 150 200 250
0 50 100 150 200 250

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