Exploiting Self-Similarity For Change-Detection

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THE MOTIVATING IDEA

...and our contribution

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- Detecting changes in the data-generating process is very important as these might indicate out of control states
 - Faults in the sensing apparatus
 - Anomalous operating conditions
 - Environmental Changes





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- 1. Fit a model / build a predictor for the time series $f_{\widehat{\theta}}(t)$
- 2. For each incoming samples compute the residuals

$$e(t) = s(t) - f_{\widehat{\theta}}(t)$$

3. Monitor the stationarity of the residuals



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- 3. Monitor the stationarity of the residuals







- Unfortunately, it is sometimes difficult to:
 - find good model family (i.e., f)
 - reliably fit this model (i.e., estimating $\hat{\theta}$)

Often, signals and time series are redundant and exhibit self-similarity



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 Often, signals and time series are redundant and exhibit self-similarity



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 The process, in its normal state, exhibits some structure which is redundant, self similar, repeated



- The process, in its normal state, exhibits some structure which is redundant, self similar, repeated
- Out of of control states instead exhibit patterns that are instead different from the normal state.





- We present a Change Detection Test (CDT) to sequentially monitor time series that uses self-similarity to
 - Characterize normal state of the process
 - Detect any departure from normal condition



- Self similarity as a powerful prior
- Problem Formulation
- Proposed Solution
 - Change Indicator
 - Search Regions
 - The Algorithm
- Experiments
- Dicussion and Conclusions



SELF SIMILARITY

A powerful prior in signal-image processing

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Self similarity is a powerful prior

- Texture completion
- Denoising (Regression)
- Inpainting (Reconstruction)



Image courtesy of Alessandro Foi http://www.cs.tut.fi/~foi/

 Never used for discriminative purposes in a sequential detection task

Self similarity as a powerful prior

- Self-similarity is measured patch-wise
- We consider 1D datastreams $\{s(\tau), \tau = 1, ...\}, s(\tau) \in \mathbb{R}$
- We define a patch centered at t having size v as

$$\mathbf{s}_{t} = \{s(t - \nu), \dots, s(t), \dots, s(t + \nu)\}$$



• The distance between two patches is the ℓ_2 norm of their difference $\|s_t - s_\tau\|_2 = \sqrt{\sum_{i=-\nu}^{\nu} (s(t+i) - s(\tau+i))^2}$



PROBLEM FORMULATION

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Problem Formulation

- Let us assume that a **process** *S* generates a **datastream** $\{s(\tau), \tau = 1, ...\}, s(\tau) \in \mathbb{R}$
 - *S* has to exhibit self similarity in the normal state
- We say that there is a change at T* if S permenently shifts from the normal state into an out of control state.
- We consider out of control states that modifies selfsimilarity of S
 - the patches from {s(τ), τ = 1, ..., T*} are not similar to patches from {s(τ), τ = T* + 1, ...}.
- Goal: Given a normal training sequence TS, detect changes analyzing, in a sequential and online manner {s(τ), τ = L + 1, ... }



PROPOSED SOLUTION

Exploiting self similarity for performing changedetection

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• We build a training set for *normal patches*

$$\mathbf{P} = \{ s_t, t = \nu, ..., M - \nu \}$$



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• We build a training set for *normal patches*

$$\mathbf{P} = \{ s_t, t = v, ..., M - v \}$$



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• We build a training set for *normal patches*

$$\mathbf{P} = \{ s_t, t = \nu, ..., M - \nu \}$$

Intuition:

$$\{ \exists s_u \in \mathbf{P} \text{ similar to } s_t, \forall t < T^* \text{ Normal} \\ \nexists s_u \in \mathbf{P} \text{ similar to } s_t, \forall t \ge T^* \text{ Out of Control}$$



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The Proposed Solution

• We build a training set for *normal patches*

$$\mathbf{P} = \{ s_t, t = \nu, ..., M - \nu \}$$

Intuition:

$$\{ \exists s_u \in \mathbf{P} \text{ similar to } s_t, \forall t < T^* \text{ Normal} \\ \exists s_u \in \mathbf{P} \text{ similar to } s_t, \forall t \ge T^* \text{ Out of Control}$$



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The Change Indicator

- We need to construct a change indicator x(t) to quantitatively assess our intuition
- We expect the change indicator x(t) to satisfy
 - {x(t), t < T*} should be i.i.d. realizations of an unknown random variable
 - {x(t), t ≥ T*} should come from a different distribution, not necessarily being i.i.d.
- Out of control states can be detected as changes in the distribution of x
 - We can use any statistical process control technique

The Change Indicator (cnt.)

- The compute the change indicator x(t) we first identify the most similar patch in P to s_t.
- We define $\pi(\cdot)$ as the map that associate to t the location $\pi(t)$ of the patch **P** of that is most similar to s_t

$$\pi(t) = \underset{\tau=\nu,\dots,M-\nu}{\operatorname{argmin}} || \mathbf{s}_t - \mathbf{s}_\tau ||_2$$

the values of $\pi(\cdot)$ can be com

• x(t) is the difference between the centers of s_t and $s_{\pi(t)}$

$$x(t) = s(t) - s(\pi(t))$$

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In Ideal Conditions

- Assume perfect matches in normal conditions, i.e., s_t and $s_{\pi(t)}$ differ only because of noise
- Then, $\forall t < T^*$

$$x(t) = s(t) - s(\pi(\tau)) = \eta$$

i.i.d random variable and $E[\eta] = 0$

- While ∀t > T*, we do not expect perfect matches: some bias appears in x(t), namely E[x(t)] ≠ 0
- In this case, it is possible to detect changes in x(t) by means of any sequential CDT.



- In the real life, perfect matches are rare
 - Patches do not differ only because of noise
 - Noise affects also the association function $\pi(\cdot)$
- However, there is an experimental evidence that patch similarity well correlates with the similarity between their central pixels
 - This is the idea behind Non Local Means filter [Buades et al 2005], which introduced a well established paradigm in signal/image processing

[Buades et al 2005] A. Buades, B. Coll, and J. Morel, "A review of image denoising algorithms, with a new one," Multiscale Modeling Simulation, vol. 4, no. 2, p. 490, 2005.

The Search Region

 Often, self similarity is due to periodic or cyclic nature of the phenomenon under monitoring



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 Often, self similarity is due to periodic or cyclic nature of the phenomenon under monitoring



- Search for similar patches should be constrained to the same time instants accross the periods
 - This determines what «out of control states» are
 - This improves computational complexity



• The function $\pi(\cdot)$ is thus defined

$$\pi(t) = \underset{\tau \in R_{\phi,t,\delta}}{\operatorname{argmin}} || \mathbf{s}_t - \mathbf{s}_\tau ||_2$$

• Being, $R_{\phi,t,\delta} = \bigcup_i \{\tau, \text{ s.t. } |t_0 + i\phi - \tau| < \delta \}$





























pixel-wise distance



The CDT on the change indicators

- A CDT can be used to detect online and sequentially, changes in the distribution of x.
- CDTs often require a training sequence containing values of x that have been computed when S is in the normal state
- Change indicators are monitored by a CDT
 - We used the ICI-based CDT [Alippi et al 2010]

[Alippi et al 2010] C. Alippi, G. Boracchi, and M. Roveri, "A Just-In-Time adaptive classification system based on the Intersection of Confidence Intervals rule," Neural Networks, vol. 24, no. 8, pp. 791 – 800, 2011.

The Algorithm: The Training Phase

- CDTs often require a training sequence of values of x, computed when S is in the normal state
- Change indicators are monitored by a CDT
 - We used the ICI-based CDT [Alippi et al 2010]
- The initial training set *TS* is divided in two parts



The CDT Training Set





Build the set of training patches **P**

1-	input : $\{s(\tau), \tau = 1,, L\}, \nu, \delta, \phi, M$						
2-	define P from $TS = \{s(\tau), \tau = 1,, M\}$ as in (3),						
3-	for $(t = M + 1; t \le L; t++)$ do						
4-	extract the patch s_t as in (1),						
5-	define the search region $R_{t,\phi,\delta}$ as in (8),						
6-	compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7)						
7-	compute the change indicator $x(t)$ as in (6),						
	end						
8-	configure the CDT on $\{x(t), t = M + 1, \dots, L\},\$						
9-	wait for the next ν samples,						
10-	while $(s(t + \nu) \text{ arrives})$ do						
11-	extract the patch s_t as in (1),						
12-	define the search region $R_{t,\phi,\delta}$ as in (8),						
13-	compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7)						
14-	compute the change indicator $x(t)$ as in (6),						
15-	if $(CDT(\{x(\tau), \tau = M,, t\}) == 1)$ then						
16-	detect a structural change in S at $\widehat{T} = t$.						
17-	return.						
	end						
18-	t = t + 1;						
	end						

Training Phase



Compute the change indicators over normal data

1-	input : $\{s(\tau), \tau = 1,, L\}, \nu, \delta, \phi, M$
2-	define P from $TS = \{s(\tau), \tau = 1, \dots, M\}$ as in (3),
3-	for $(t = M + 1; t \le L; t++)$ do
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Configure the ICIbased CDT on these change indicators

input: $\{s(\tau), \tau = 1, ..., L\}, \nu, \delta, \phi, M$ 1define **P** from $TS = \{s(\tau), \tau = 1, ..., M\}$ as in (3), 2for $(t = M + 1; t \le L; t++)$ do 3-4extract the patch s_t as in (1), 5define the search region $R_{t,\phi,\delta}$ as in (8), 6compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) 7compute the change indicator x(t) as in (6), end configure the CDT on $\{x(t), t = M + 1, \dots, L\}$, 8-9wait for the next ν samples, 10while $(s(t + \nu) \text{ arrives})$ do 11extract the patch s_t as in (1), define the search region $R_{t,\phi,\delta}$ as in (8), 12-13compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) 14compute the change indicator x(t) as in (6), if $(CDT(\{x(\tau), \tau = M, ..., t\}) == 1)$ then 15detect a structural change in S at $\hat{T} = t$. 16-17return. end t = t + 1;18end

1	The Algorithm		
Operational Life	Crop a patch around $s(t)$	1- 2- 3- 4- 5- 6- 7- 8- 9- 10- 11- 12- 13- 14- 15- 16- 17- 18-	input: $\{s(\tau), \tau = 1,, L\}, \nu, \delta, \phi, M$ define P from $TS = \{s(\tau), \tau = 1,, M\}$ as in (3), for $(t = M + 1; t \le L; t++)$ do extract the patch s_t as in (1), define the search region $R_{t,\phi,\delta}$ as in (8), compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7 compute the change indicator $x(t)$ as in (6), end configure the CDT on $\{x(t), t = M + 1,, L\}$, wait for the next ν samples, while $(s(t + \nu) \text{ arrives})$ do extract the patch s_t as in (1), define the search region $R_{t,\phi,\delta}$ as in (8), compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7 compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7 compute the change indicator $x(t)$ as in (6), if $(CDT(\{x(\tau), \tau = M,, t\}) == 1)$ then detect a structural change in S at $\widehat{T} = t$. return. end t = t + 1;
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2	The Algorithm		
Operational Life	Compute the change indicator $x(t)$	1- 2- 3- 4- 5- 6- 7- 8- 9- 10- 11- 12- 13- 14- 15- 16- 17- 18-	input: $\{s(\tau), \tau = 1,, L\}, \nu, \delta, \phi, M$ define P from $TS = \{s(\tau), \tau = 1,, M\}$ as in (3), for $(t = M + 1; t \le L; t++)$ do extract the patch s_t as in (1), define the search region $R_{t,\phi,\delta}$ as in (8), compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) compute the change indicator $x(t)$ as in (6), end configure the CDT on $\{x(t), t = M + 1,, L\}$, wait for the next ν samples, while $(s(t + \nu) \text{ arrives})$ do extract the patch s_t as in (1), define the search region $R_{t,\phi,\delta}$ as in (8), compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) compute the patch most similar to s_t in $R_{t,\phi,\delta}$, (7) compute the search region $R_{t,\phi,\delta}$ as in (8), compute the search region $R_{t,\phi,\delta}$ as in (6), if $(CDT(\{x(\tau), \tau = M,, t\}) == 1)$ then detect a structural change in S at $\widehat{T} = t$. return. end t = t + 1;
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Operational Life

Run the CDT until a change is detected

The Algorithm

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input: $\{s(\tau), \tau = 1, ..., L\}, \nu, \delta, \phi, M$

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return.

end

t = t + 1;

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15-

16-

17-

18-



EXPERIMENTS

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- Flow measured in Barcelona Water Distribution Networks
 - Measurements from different DMA inlets
 - One measure every 10 minutes, daily period





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- 10 sequences synthetically adding a change after 41 days
 - **Offset:** $s(t) = s(t) + o, t > T^*, o \in \{0.25a, 0.5a\}$





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• Stack-at: $s(t) = k, t > T^*$



• **Residual-based:** a predictive model $f_{\hat{\theta}}$ of a nonlinear ARX (wavelet network) is used to compute

$$r(t) = s(t) - f_{\widehat{\theta}}(t)$$

The Considered CDTs

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• **Template-based:** compute the difference w.t.r template, i.i.e the average flow profile in the *n* periods in the TS



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$$r(t) = s(t) - f_{\widehat{\theta}}(t)$$

• **Template-based:** compute the difference w.t.r template, i.i.e the average flow profile in the *n* periods in the TS

$$p(t) = s(t) - \frac{1}{n} \sum_{i=1}^{n} s(t_0 + i\phi)$$

Self-similarity: the proposed solution, v = 5, $\delta = 5$

Details:

2 weeks of recordings used for bulding **P** / model fitting / template estimation, 400 samples for CDT configuration

Change indicators in normal conditions

 The autocorrelation of the considered change indicators in normal conditions



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Change Detection Performance

- FPR: False Positive Rate
- FNR: False Negative Rate
- DD: Expected Detection Delay

	Self-Similarity based			Residuals-based			Template-based		
	FPR	FNR	DD	FPR	FNR	DD	FPR	FNR	DD
offset 0.5	0.1	0.0	156.4	0.0	0.0	1554.0	0.2	0.0	332.0
offset 0.25	0.1	0.0	914.2	0.0	0.3	2803.4	0.2	0.0	747.0
sensor degradation 0.5	0.1	0.0	174.2	0.0	0.0	170.0	0.2	0.0	269.5
sensor degradation 0.25	0.1	0.0	336.4	0.0	0.0	288.0	0.2	0.0	652.0
source change	0.1	0.0	103.1	0.0	0.0	800.0	0.2	0.0	219.5
stack-at	0.1	0.0	169.8	0.0	0.0	160.0	0.2	0.0	534.5

Offset of +50% the average flow value



Offset of +25% the mean flow (False Positive)

CONCLUDING REMARKS

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Concluding Remarks

- Self similarity seems a promising approach for detecting changes in the structure of a self-similar datastream
 - Detection performance and autocorrelation show that x is very good at assessing self similarity
 - Detection performance indicates that x reliably reacts to changes
- Ongoing Works
 - Investigating different change indicators for assessing self similarity.
 - Exolploiting self similarity in a collaborative manner (multichannel observations)
 - Self similarity when data are not periodic
 - Automatic criteria to identify the best patch size

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Codes will be soon available for download at http://home.deib.polimi.it/boracchi/Projects/

