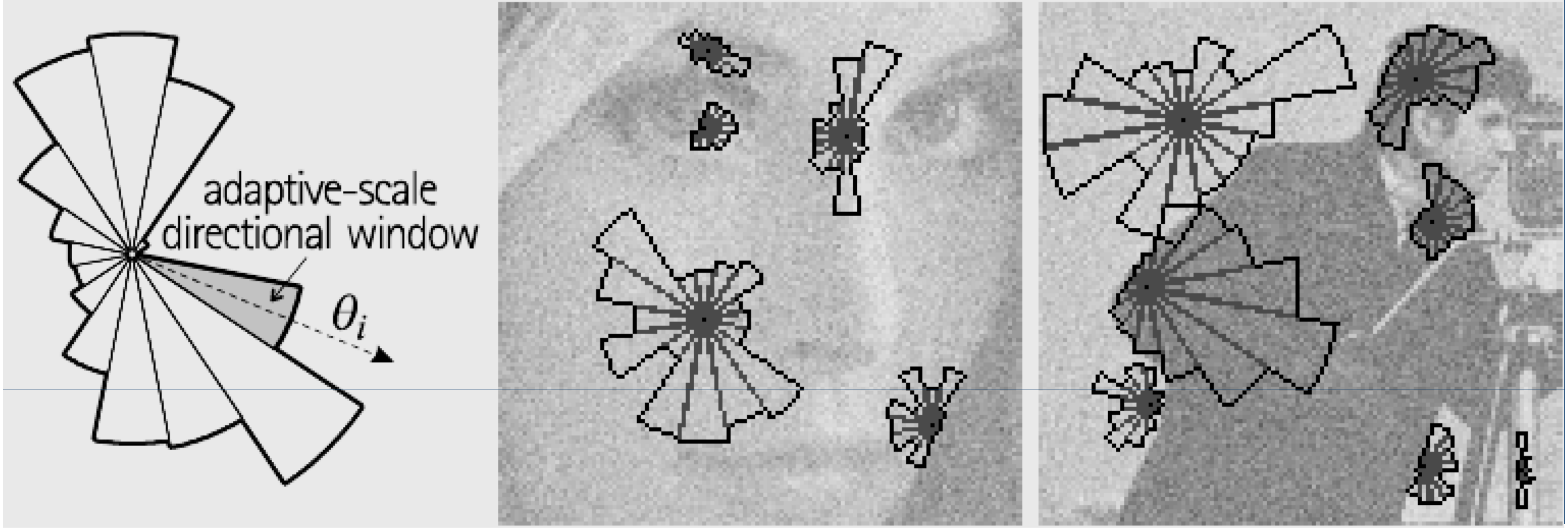

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Image Processing case study: LPA-ICI Denoising

27th October 2009

Disclaimer

- Most of the material shown in these slides is taken from *LASIP (Local Approximation in Signal and Image Processing)* laboratory webpage.

- http://www.cs.tut.fi/~lasip/

- There you can download papers and Matlab sources for most of implemented algorithms

Outline

- Image Formation Model
- Denoising approaches
- LPA-ICI Denoising Motivations
- Local Polynomial Approximation
- ICI rule
- Algorithm Details

Image Formation Model

- Observation model is

$$z(x) = y(x) + \eta(x)$$

z	Sensed image
x	Pixel index $x \in X$
y	Original (unknown) image
η	noise.

- For the sake of simplicity we will assume $\eta \sim N(0, \sigma^2)$
- The goal is to obtain $\widehat{y}(x)$, a reliable estimate of $y(x)$, given $z(x)$ and the distribution of η .

Denoising Approaches

- Parametric Approaches
  - Transform Domain Filtering, they assume the noisy-free signal is somehow sparse in a suitable domain (e.g Fourier, DCT, Wavelet) or w.r.t. some dictionary based decomposition)

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Estimating $y(x)$ from $z(x)$ can be statistically treated as regression of z on x

$$\widehat{y}(x) = E[z|x]$$

Non parametric approaches

- **Local / Non Local**

  - *Local methods: weights used for the algorithm depends on the distance between the estimation point* x_0 *and the other observation points* $x_s, \; \|x_0 - x_s\|$

Non parametric approaches

- **Local / Non Local**

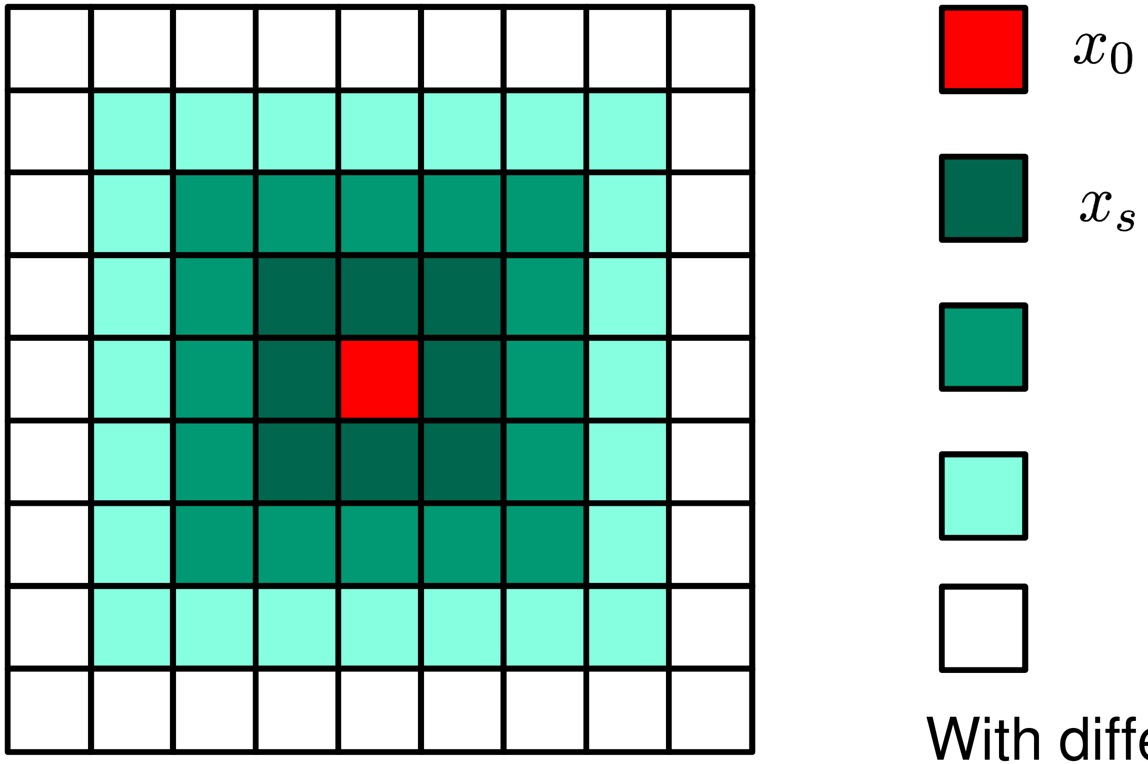
  - *Local methods: weights used for the algorithm depends on the distance between the estimation point* x_0 *and the other observation points* $x_s, \; \|x_0 - x_s\|$

  - *Non Local Methods: the weights are function of the differences of the corresponding signals. The weight used to estimate* y_0 *depends on* y_s *. Typically on something related to* $\|y_0 - y_s\|_2$ *.*

Katkovnik, V., A. Foi, K. Egiazarian, and J. Astola, "From local kernel to nonlocal multiple-model image denoising", preprint (July 2009), to appear Int. J. Computer Vision.

Local vs Non-Local

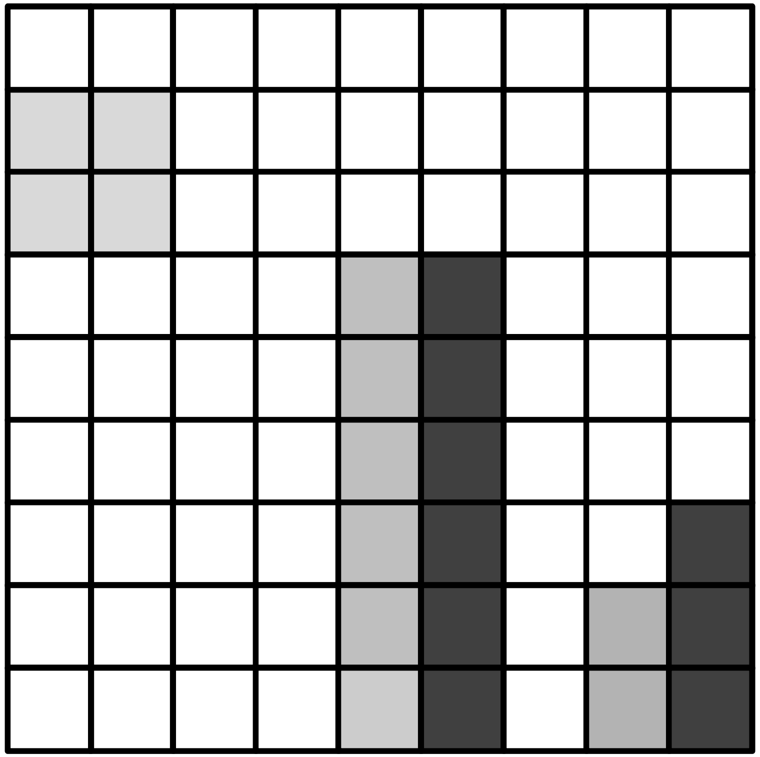
- Local, weights are determined by the pixel distance (regardless of the image content)



With different weights

Local vs Non-Local

- Non Local, weights are determined by the image similarity



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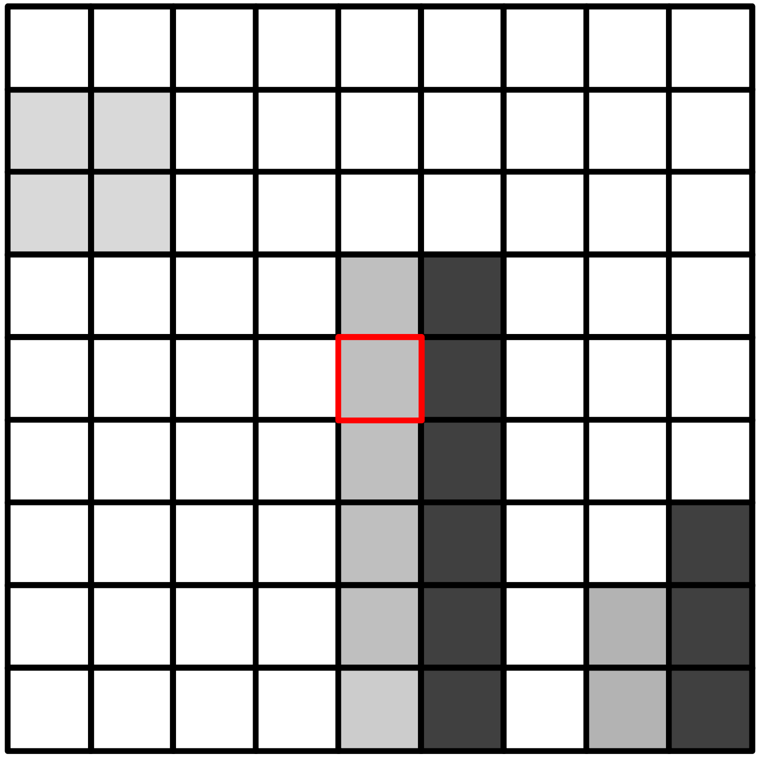

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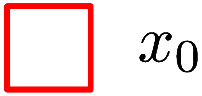

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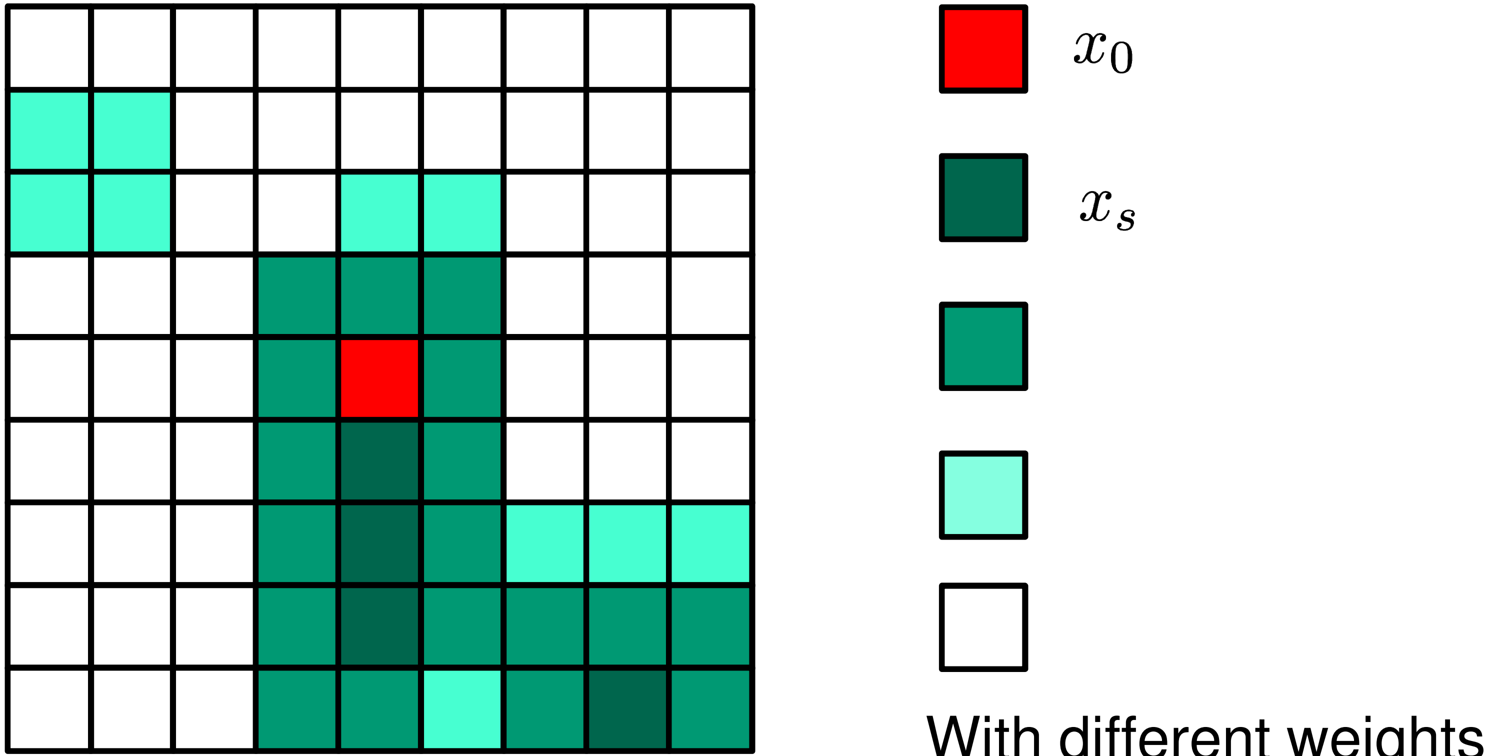

Non parametric approaches

- **Pointwise / Multipoint**

  - *Pointwise: the estimation of noise-free signal is computed for the central point only,* y_0 *and not for all the other points considered*

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  - *Pointwise: the estimation of noise-free signal is computed for the central point only,* y_0 *and not for all the other points considered*

  - *Multipoint : the estimation of the noise-free signal is computed for all the points* y_s *used by the estimator to estimate* y_0 *.*

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Pointwise vs Multipoint

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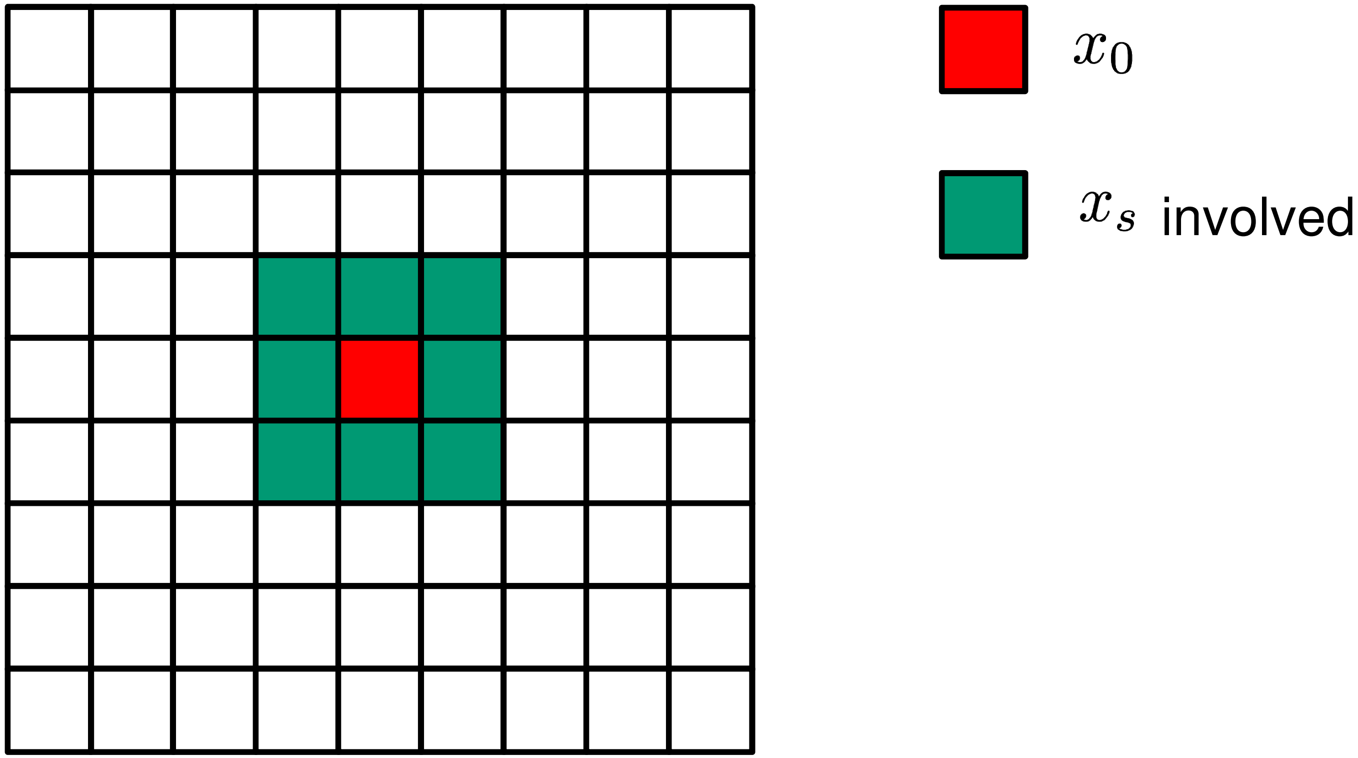

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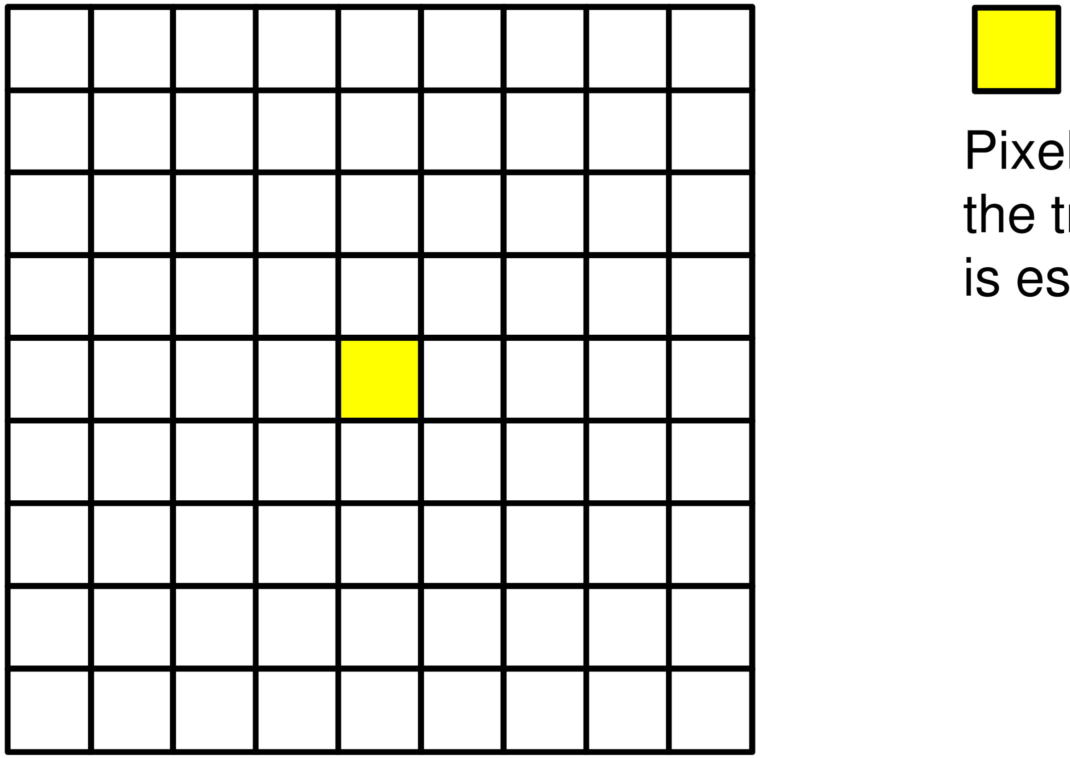

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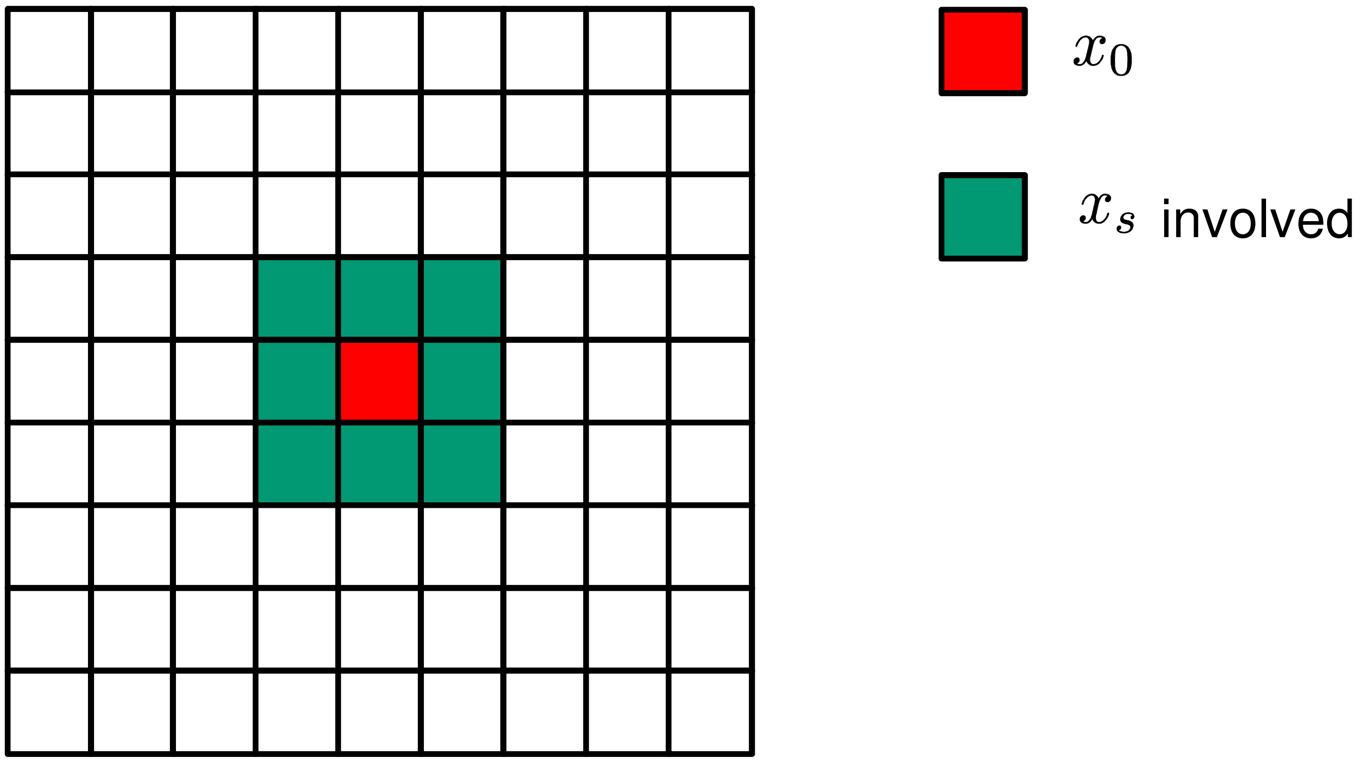

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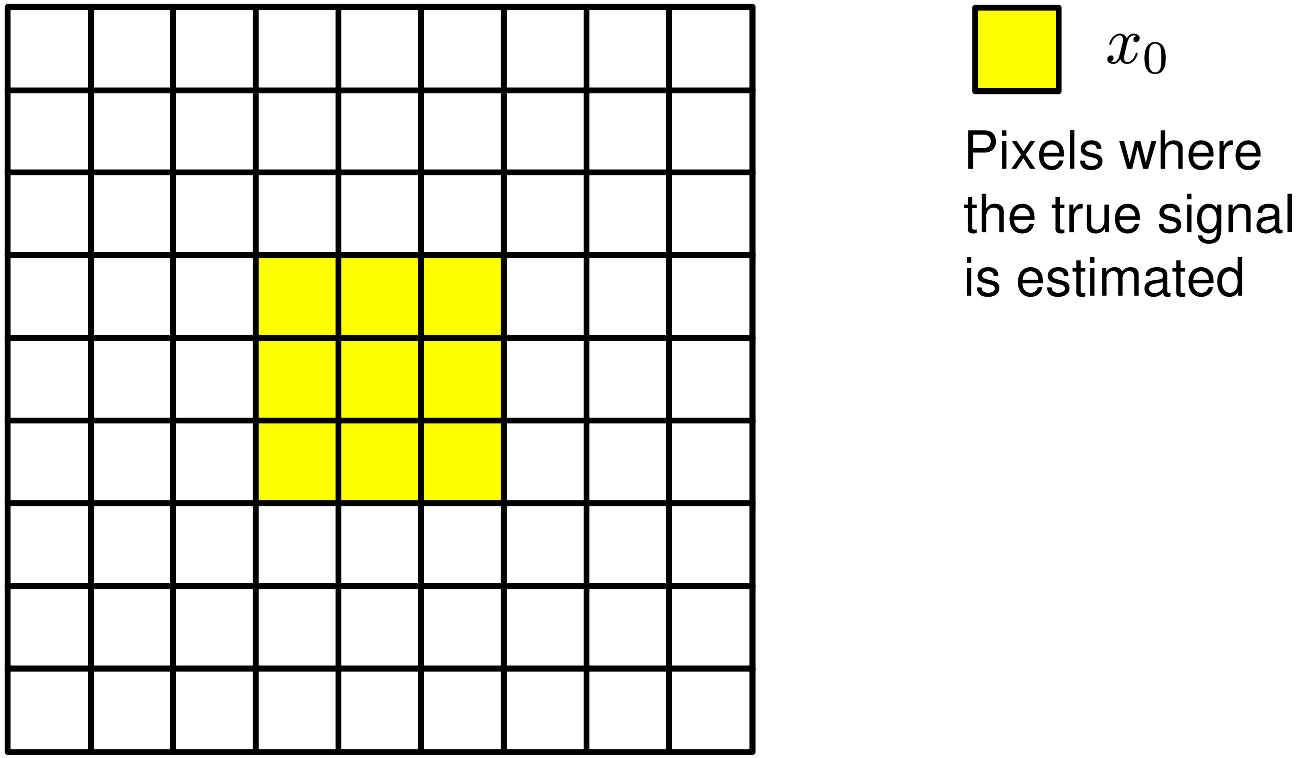

Disclaimer

- This classification holds for the principles that determine the algorithm design.

- Most of the algorithm are implemented combining methods from different approaches.

- Furthermore, some of the most effective denoising algoritms enforce parametric assumptions on image areas adatptively selected
  - SA-DCT
  - BM3D

What are we going to see then,

- A Local, pixel-wise denoising algorithm

LPA-ICI : Local Polynomial Approximation using Intersection of Confidence Interval rule.

- We will consider the **denoising** as a **basic problem**, although this method has been successfully applyied to several other problems such as
  - Deblurring
  - Interpolation
  - Enhancement
  - Demosaicing
  - Deblocking
  - Inverse Halftoning

Why?

- Why to use such an algorithm as a case study for Cuda programming?

  - It is **Embarrassingly parallel** (it is a local pixelwise method).
  - It is simple to implement.
  - It is motivated by few clear and easy-to-show assumption.
  - It has been successfully applied to **several image processing challenges**.

LPA-ICI basic ideas

- It combines two independent ideas:

  - **Local Polynomial Approximation** (LPA) for designing linear filters that performs *pixelwise polynomial fit* on a certain neighborhood.

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- It combines two independent ideas:
  - **Local Polynomial Approximation** (LPA) for designing linear filters that performs *pixelwise polynomial fit* on a certain neighborhood.
  - **Intersection of Confidence Interval** rule (ICI) is an *adaptation algorithm,* used to define the most suited neighborhood where the polynomial assumpions fit better the observations.

Local Pointwise Techniques

- **Local Pointwise weighted averages**: the estimate at x_0 is

$$\widehat{y_h}(x_0) = \sum_{x_s \in X} w_h(x_0 - x_s) z(x_s)$$

$$w_h = \{w_h(x)\} \quad s.t. \quad \sum_{x \in X} w_h(x) = 1$$

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- Local Polynomial Approximation
  - **Determine** $p_{h,m}$ the **polynomial expression** (of a fixed order *m*) that better fits the observation on a fixed pixel neighborhood w_h

$$p_{h,m} = \text{argmin}_{p \in \mathcal{P}_m} \sum_{x_s \in X} w_h(x_0 - x_s) \left(z(x_s) - p(x_s) \right)^2$$

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  - The **signal estimate** is given by $p_{h,m}(x_0)$, the **value** of this **polynomial** in x_0

$$\widehat{y_h}(x_0) = p_{h,m}(x_0)$$

the **weights** w_h determines the **localization of this fit**

LPA Kernel

- The LPA estimate can be obtained via a **convolution** with **discrete LPA kernels** g_h

$$\widehat{y_h}(x_0) = (z \circledast g_h)(x_0)$$

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- Summarizing, LPA estimates can be obtained via a convolution with **discrete kernels** which are determined by:
  - The **order** of polynomial fit.
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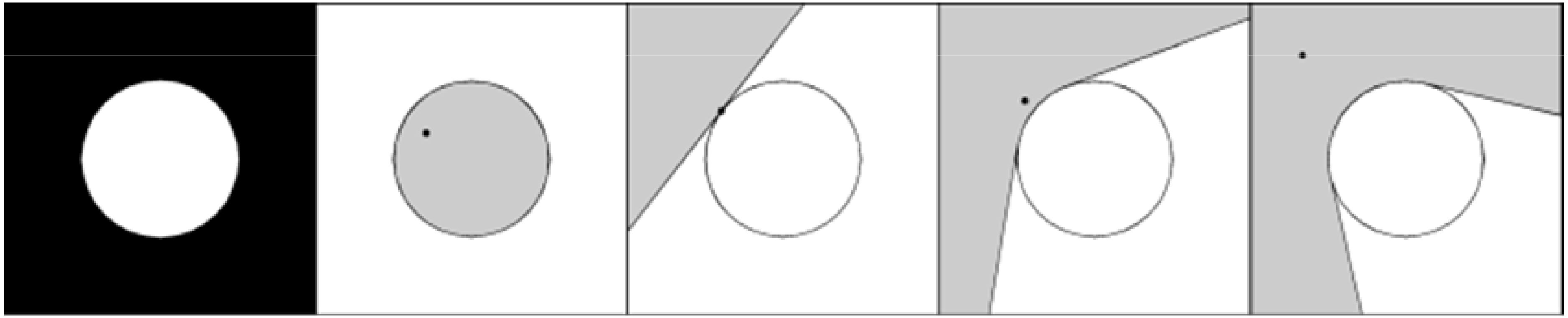
- This makes LPA a perfect tool for **designing adaptive filters**.

LPA-ICI algorithm: ideal neighborhood

- ***Ideal*** in the sense that it defines the support of **pointwise Least Square kernel estimators**.



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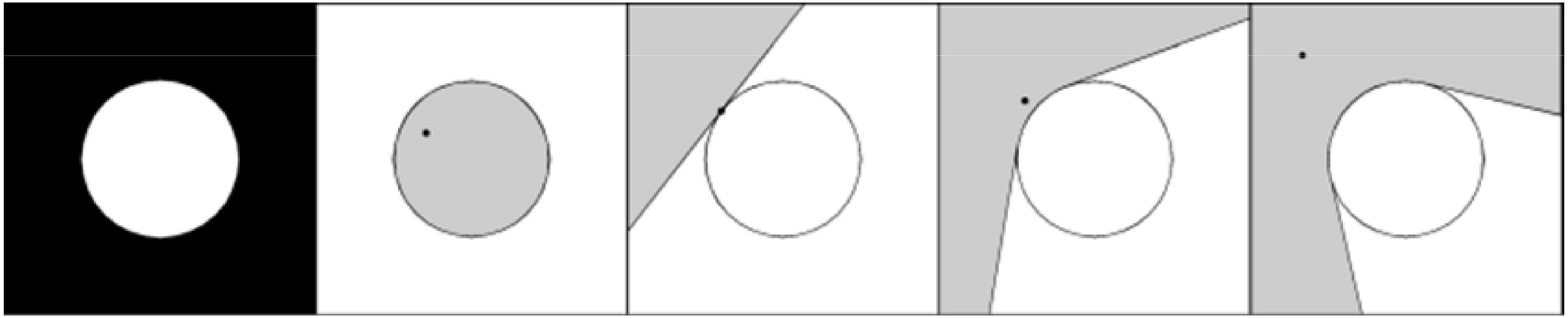

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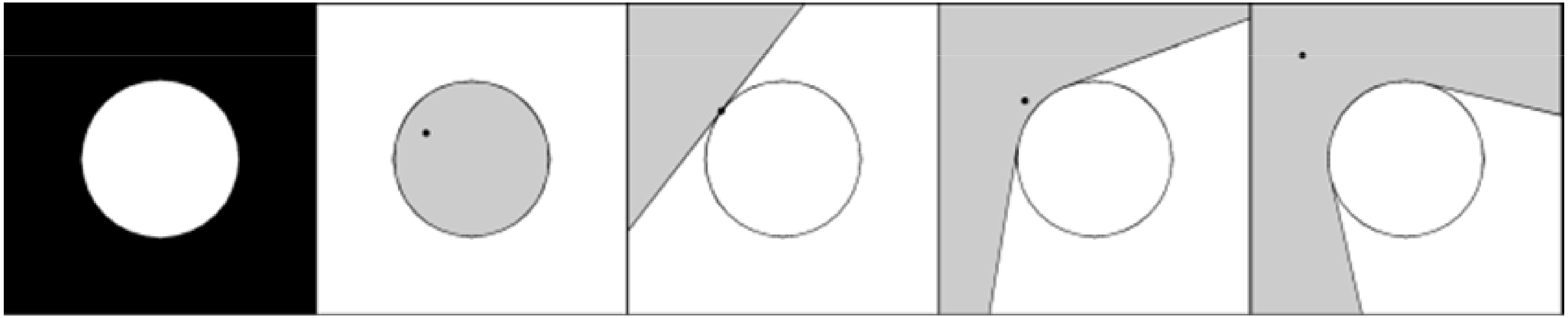

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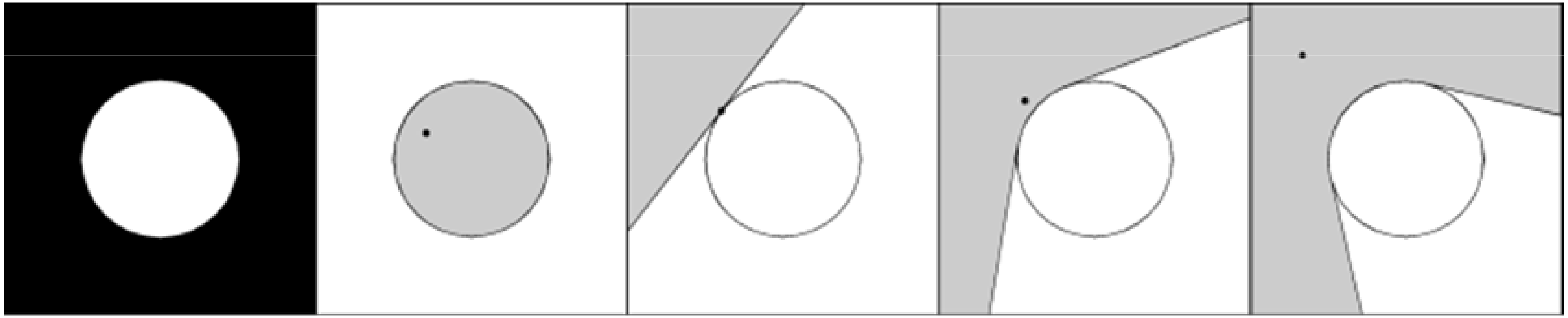

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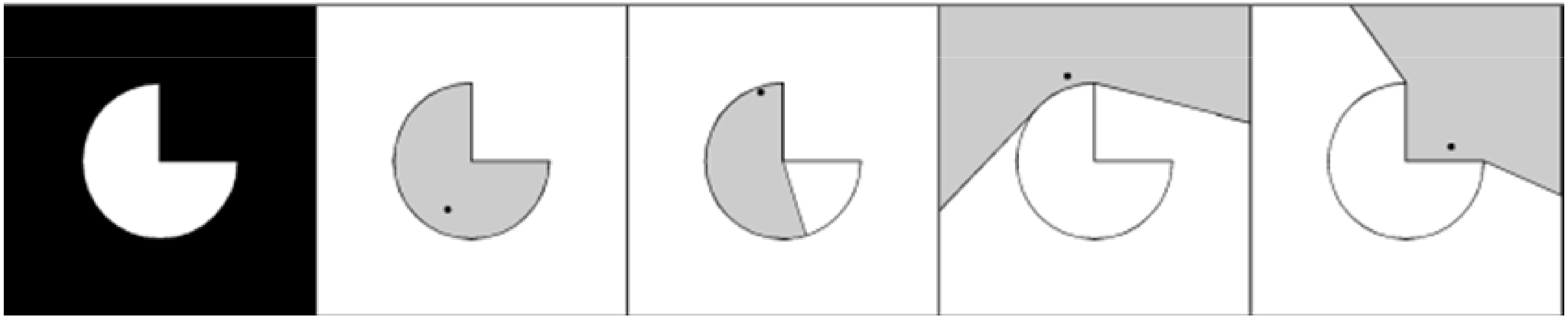
LPA-ICI algorithm: ideal neighborhood

- The ideal neighborhood is built in the discrete image domain using LPA filters having directional supports



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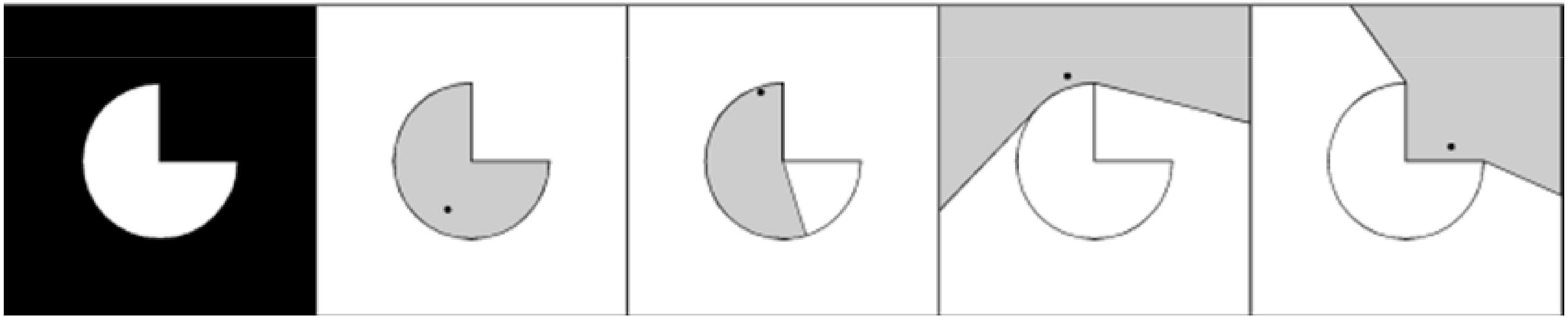
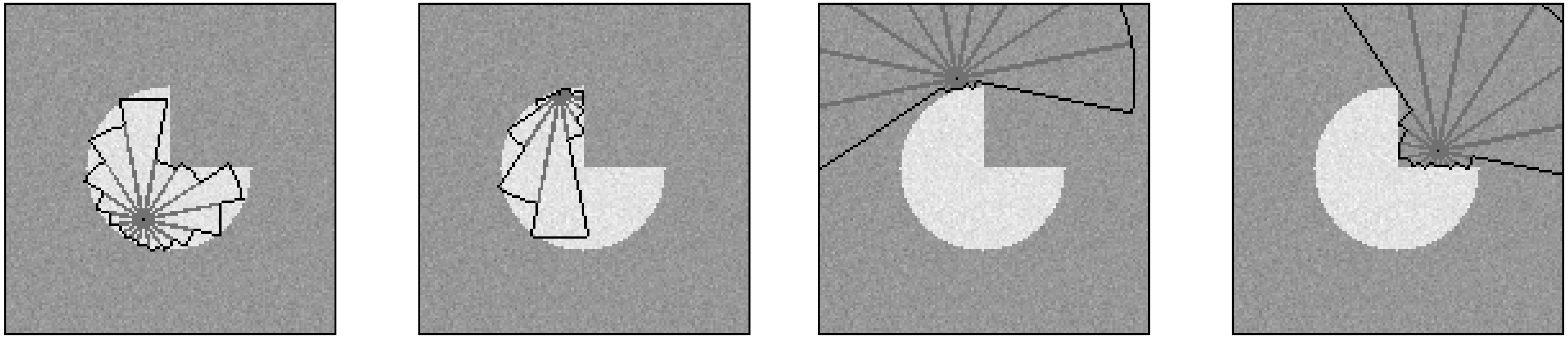
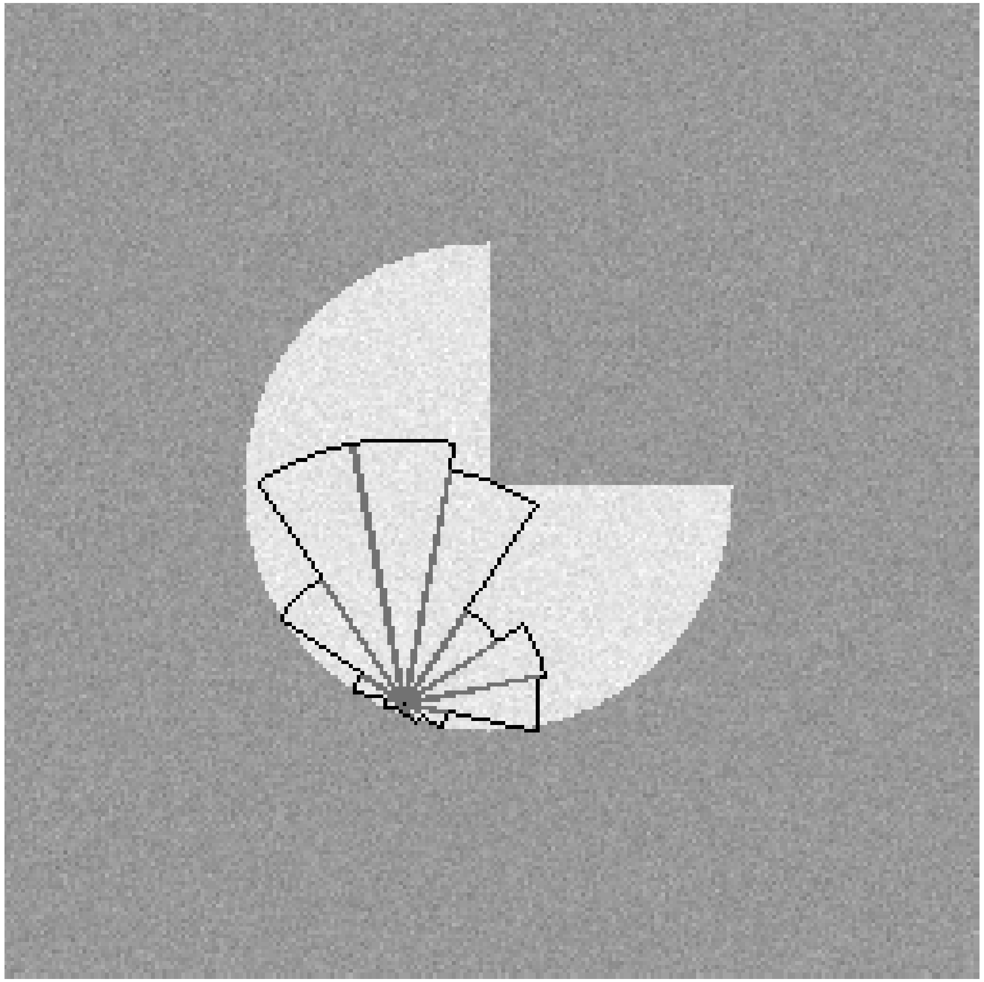




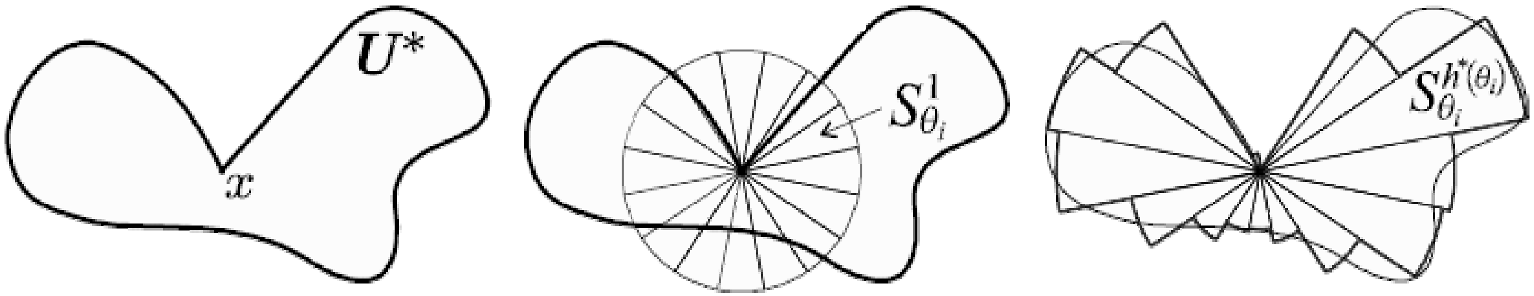
Examples of adaptively selected neighborhoods

- Adaptively selected neighborhoods selected using the LPA-ICI rule



Neighborhood discretization

- A suitable discretization of this neighborhood is obtained by using a set of directional LPA kernels $\{g_{\theta,h}\}_{\theta,h}$



Ideal Neighborhood

Directional kernels

Discrete Adaptive Neighborhood

where θ determines the orientation of the kernel support, and where h controls by the scale of kernel support.

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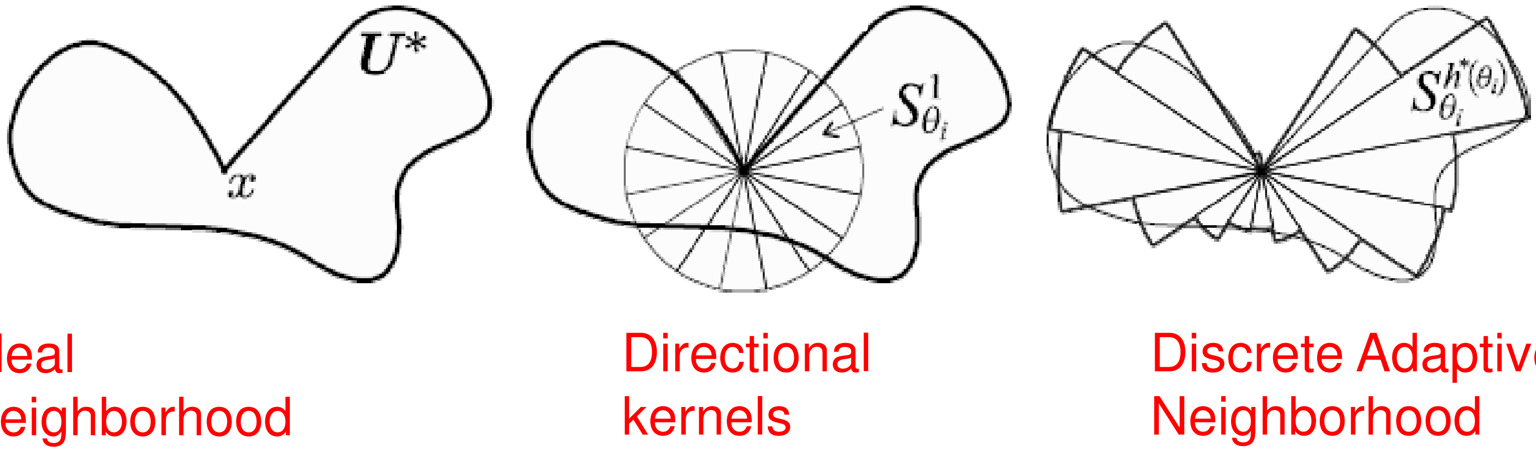
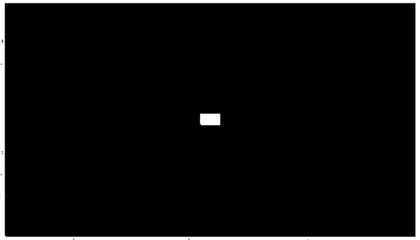


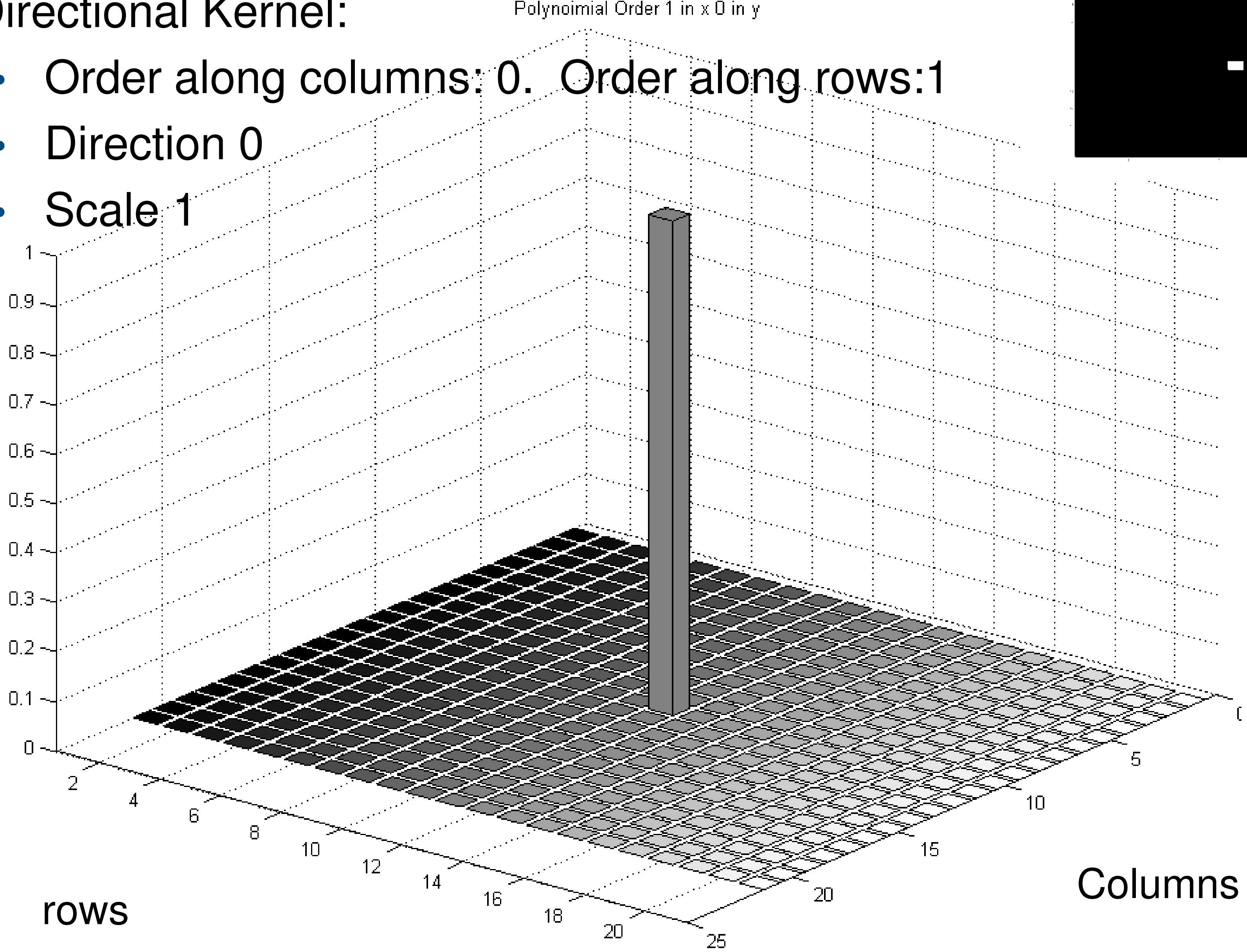
where θ determines the orientation of the kernel support, and where h controls by the scale of kernel support.

- The initial **shape optimization problem** can be solved by using standard easy-to-implement **varying-scale kernel techniques**, such as **the *ICI* rule**.

Example of LPA Anisotropic Kernels

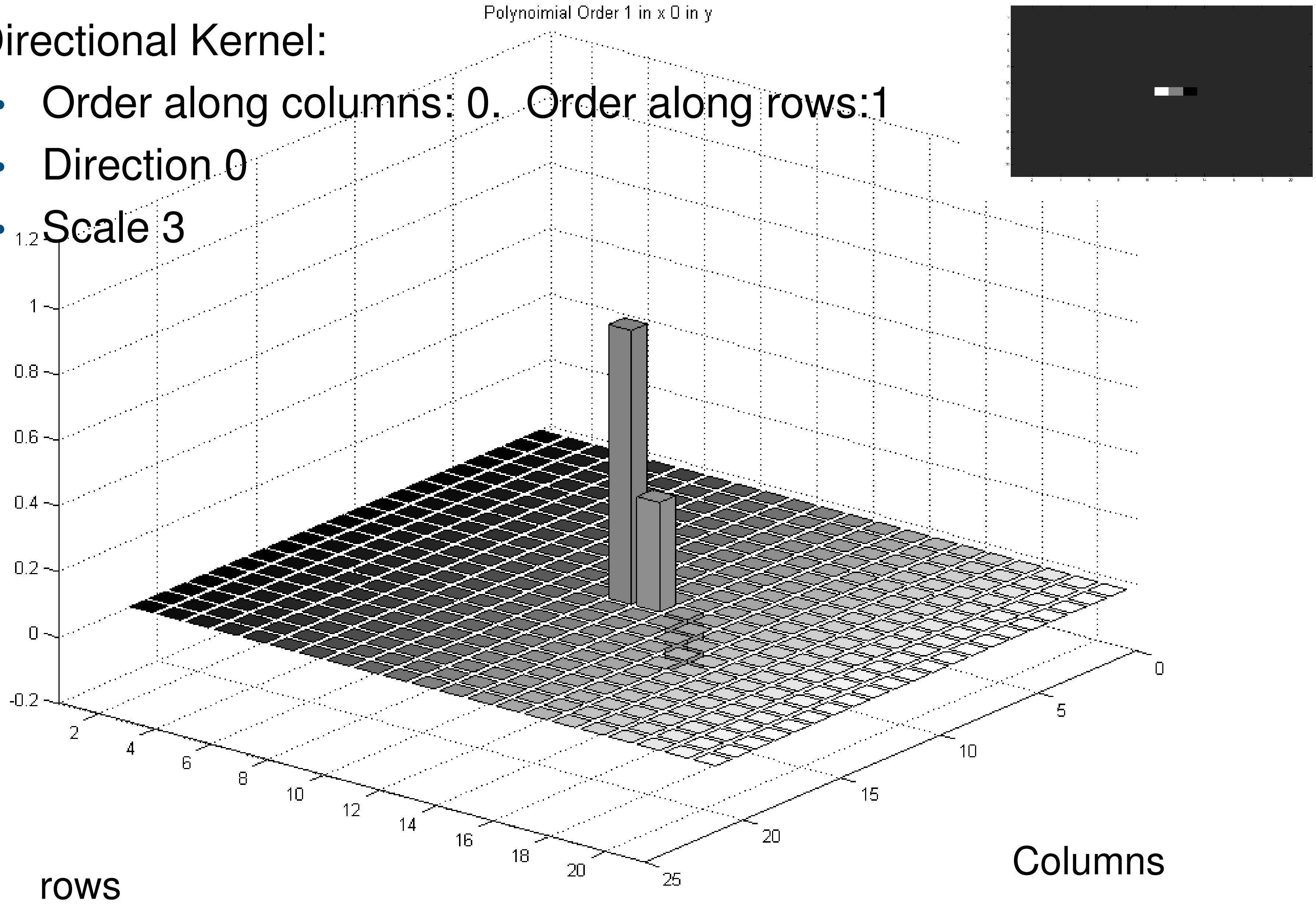
- Directional Kernel:
  - Order along columns: 0. Order along rows:1
  - Direction 0
  - Scale 1





Example of LPA Anisotropic Kernels

- Directional Kernel:
  - Order along columns: 0. Order along rows:1
  - Direction 0
  - Scale 3



Example of LPA Anisotropic Kernels

- Directional Kernel:
  - Order along columns: 0. Order along rows:1
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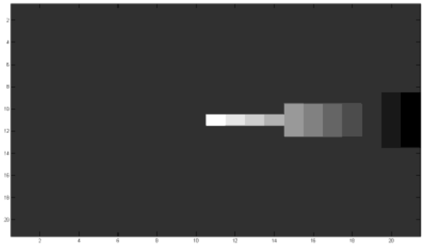
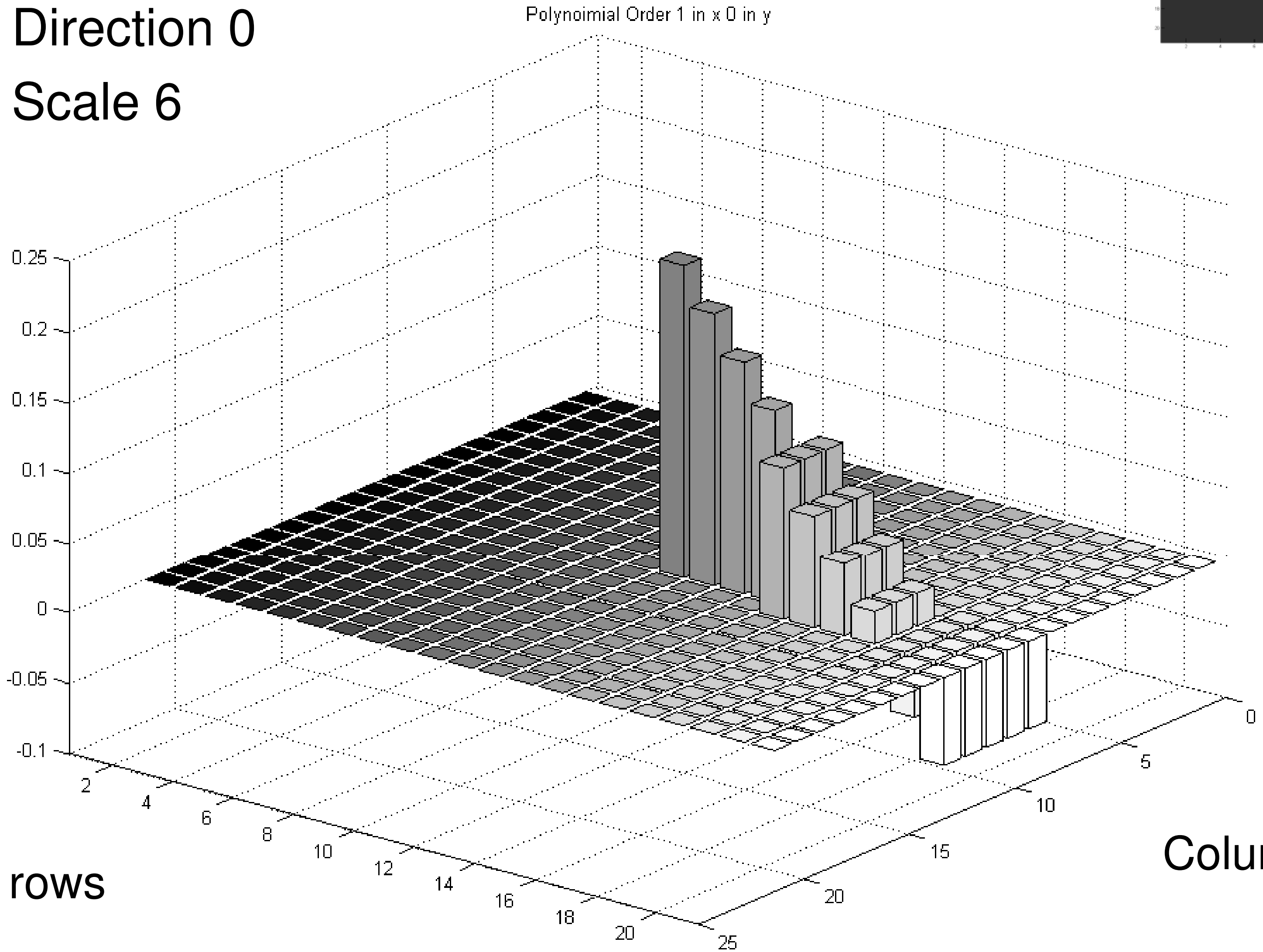
  - Scale 6





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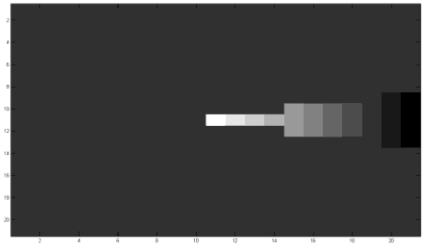
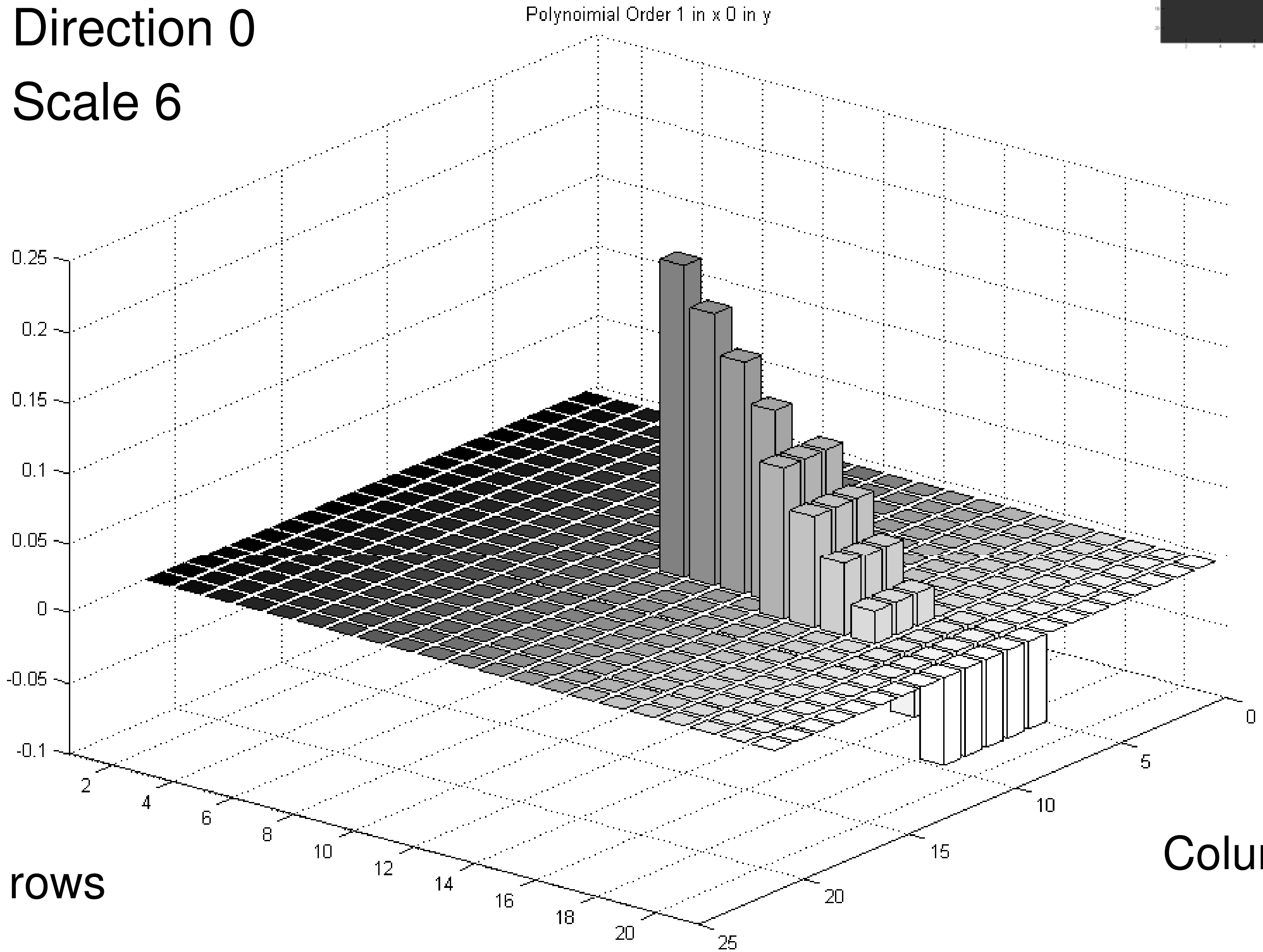
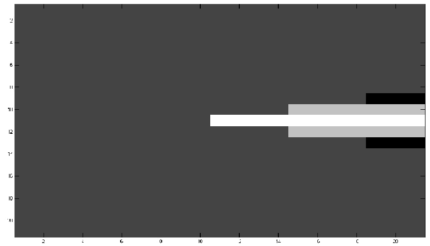
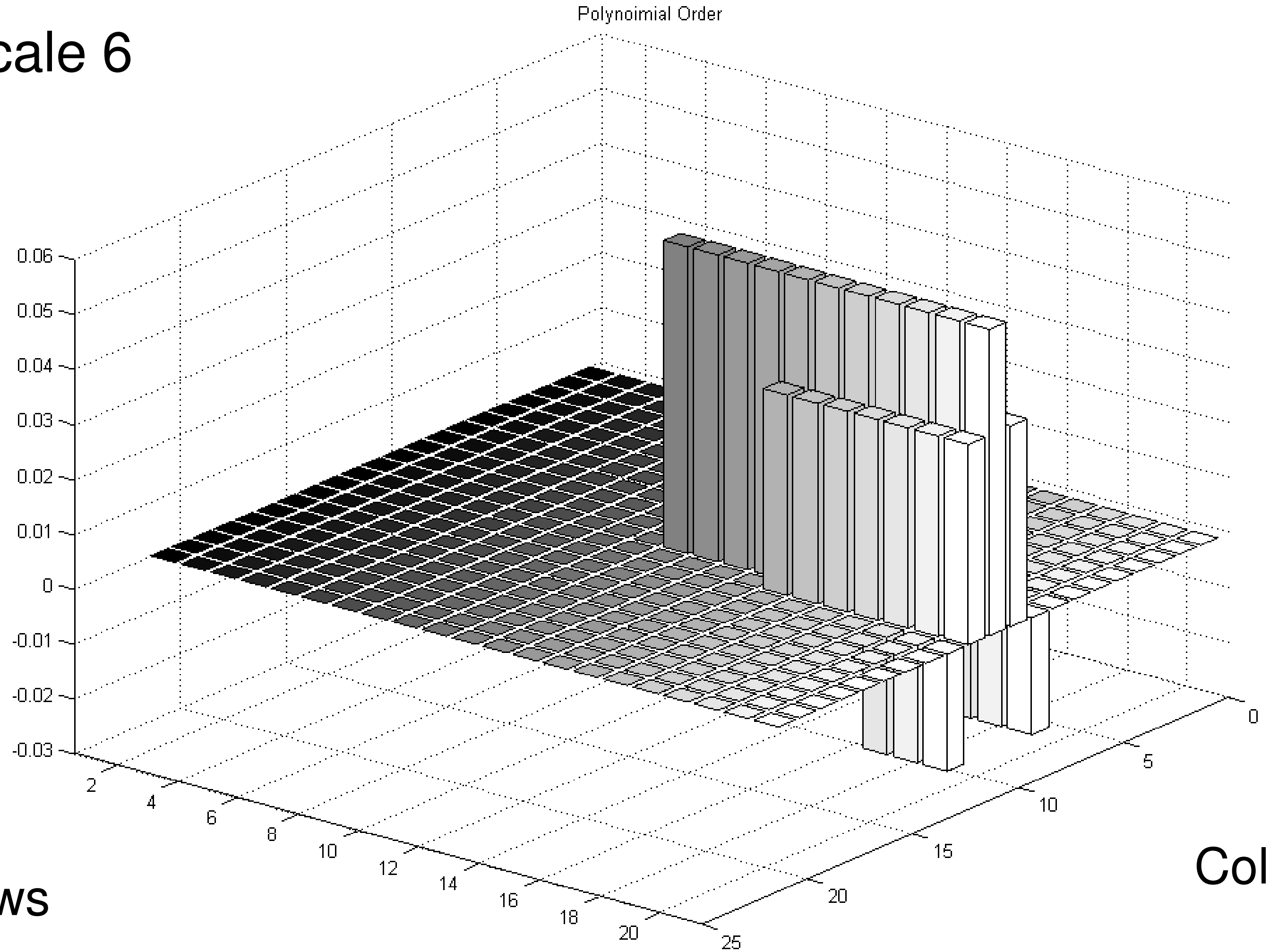




Example of LPA Anisotropic Kernels

- Directional Kernel:
  - Order along columns: 2. Order along rows: 0
  - Direction 0
  - Scale 6





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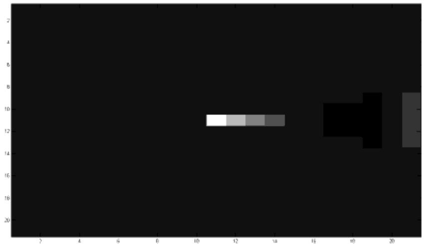

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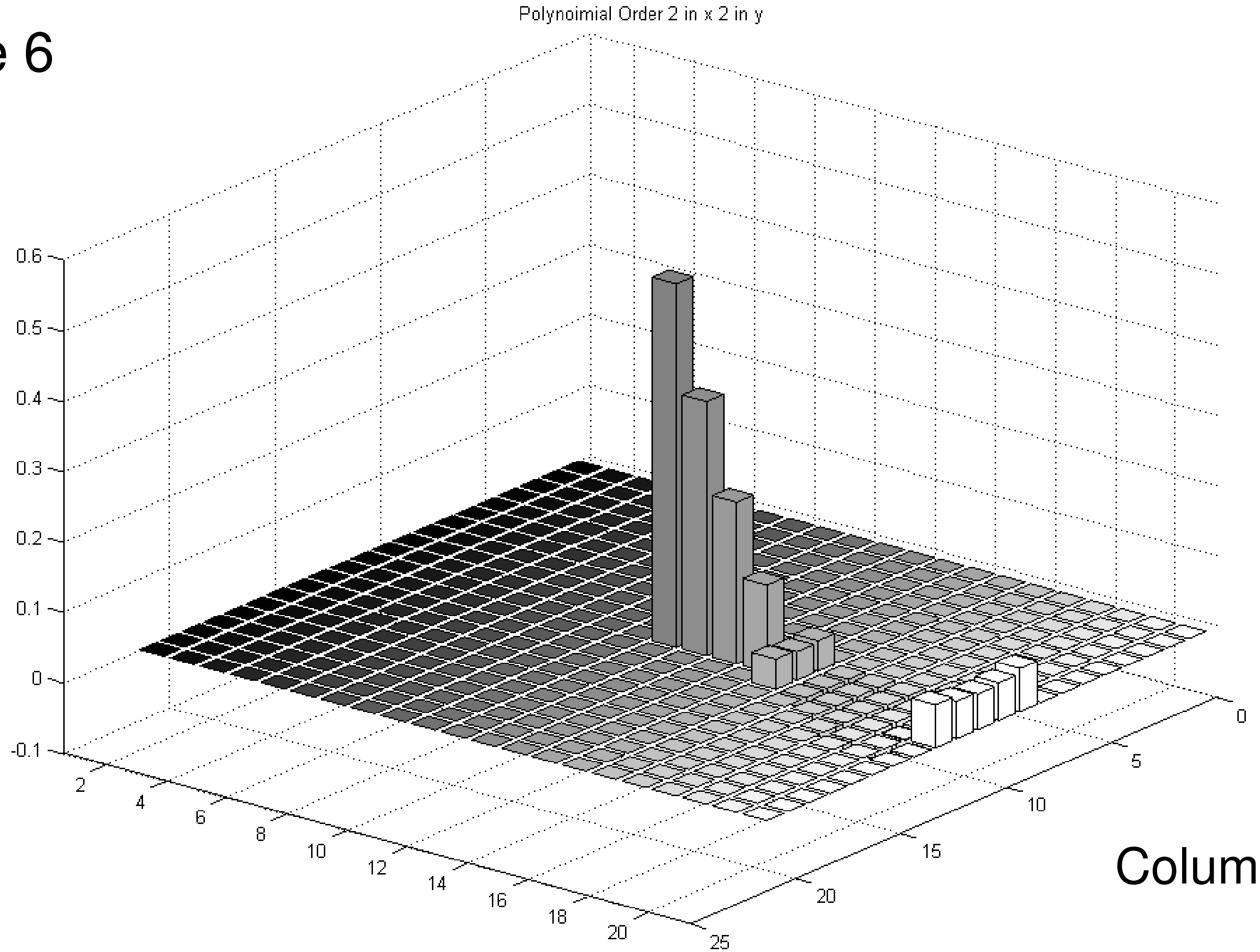
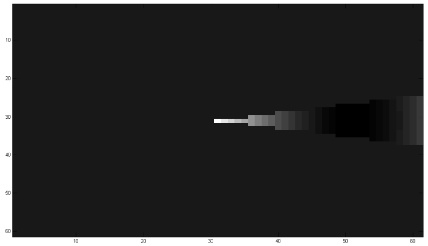
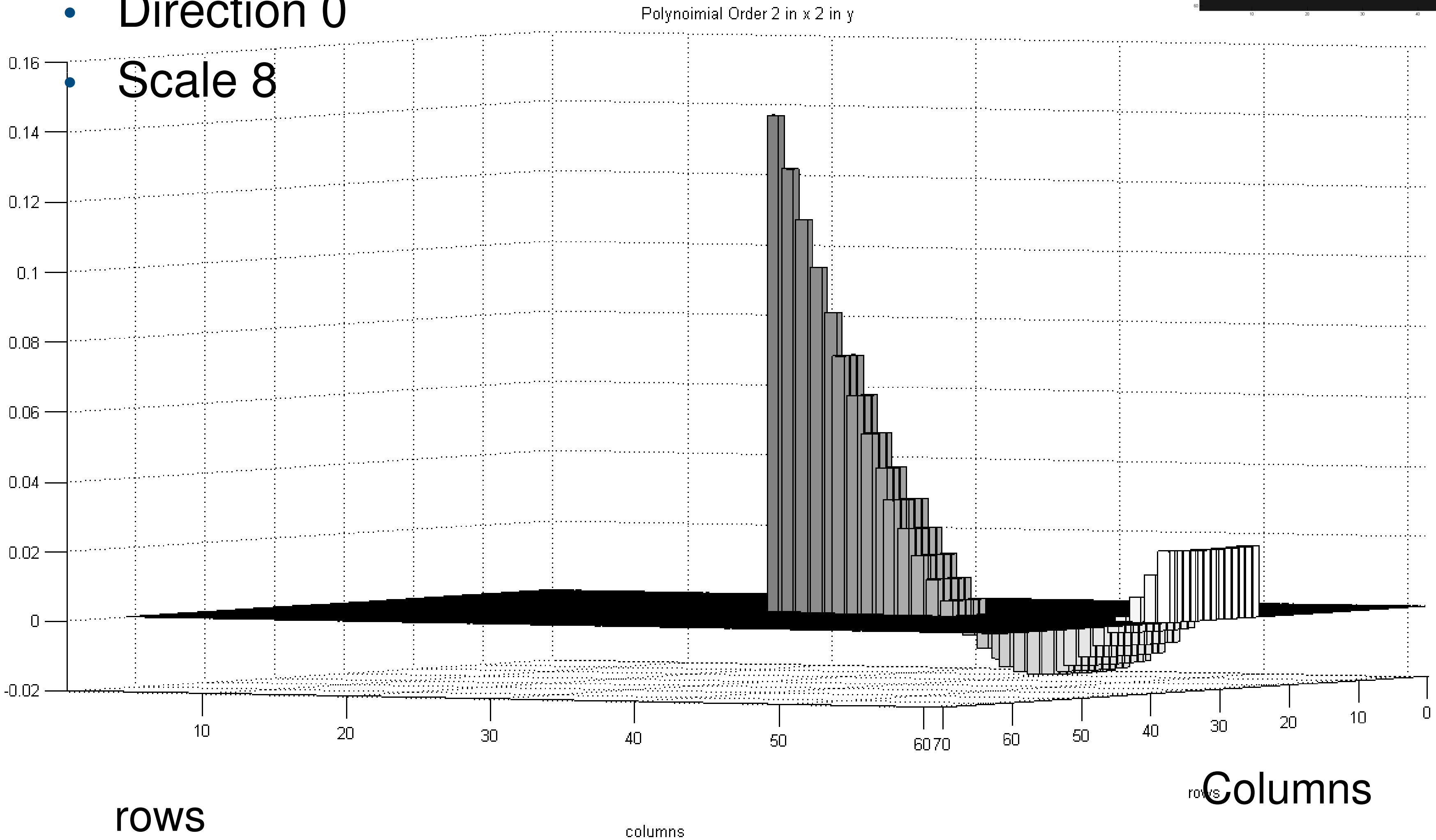


Example of LPA Anisotropic Kernels

- Directional Kernel:
  - Order along columns: 2. Order along rows: 1
  - Direction 0
  - Scale 8





Example of LPA Anisotropic Kernels

- Directional Kernel:
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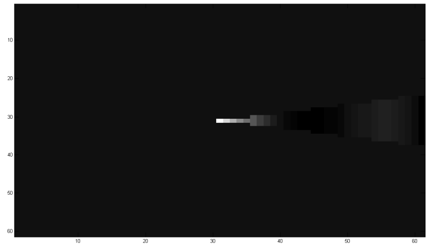
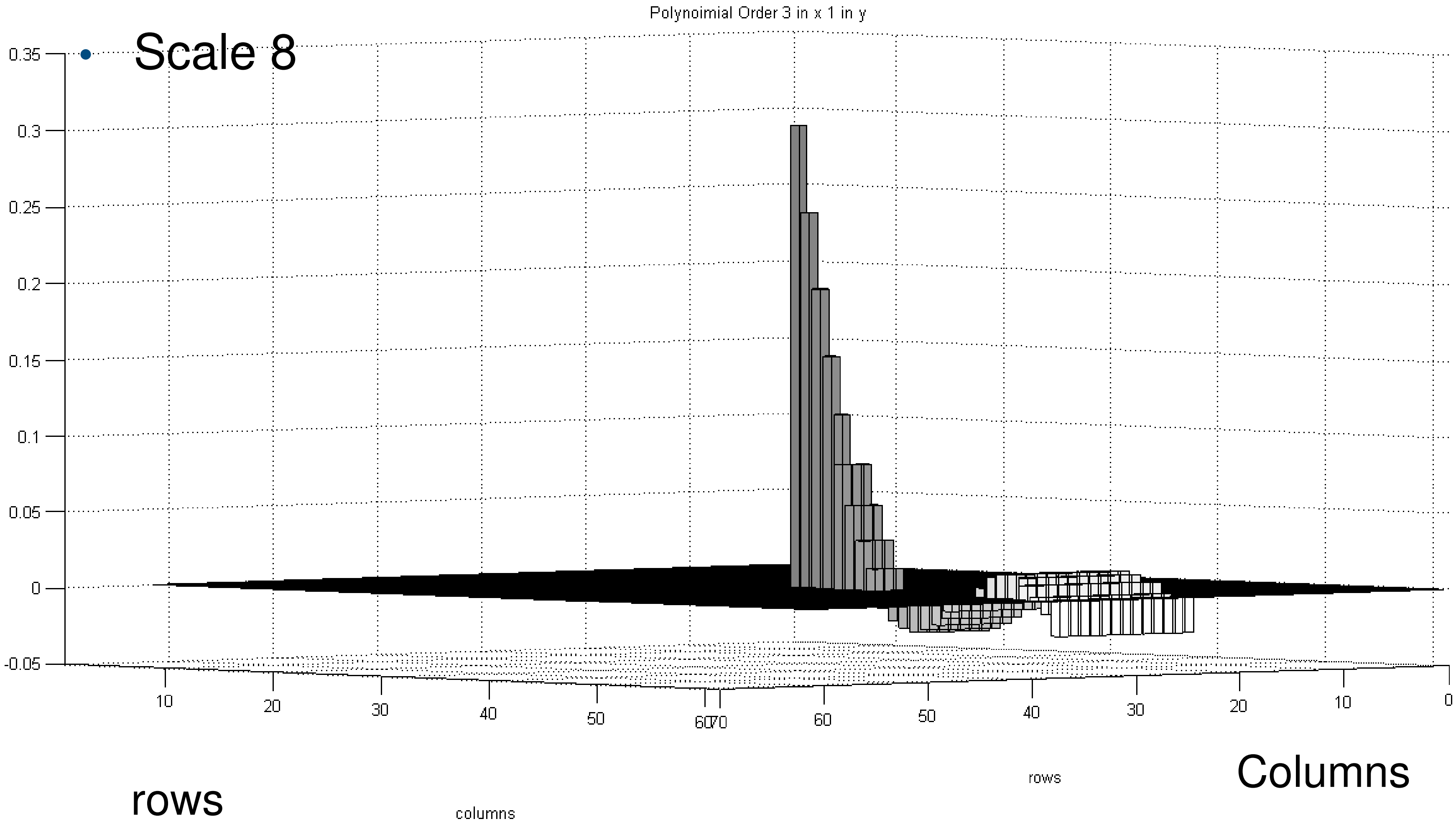
  - Scale 8





LPA Kernels for derivatives estimation

- Kernels can be defined also for polynomial derivatives:

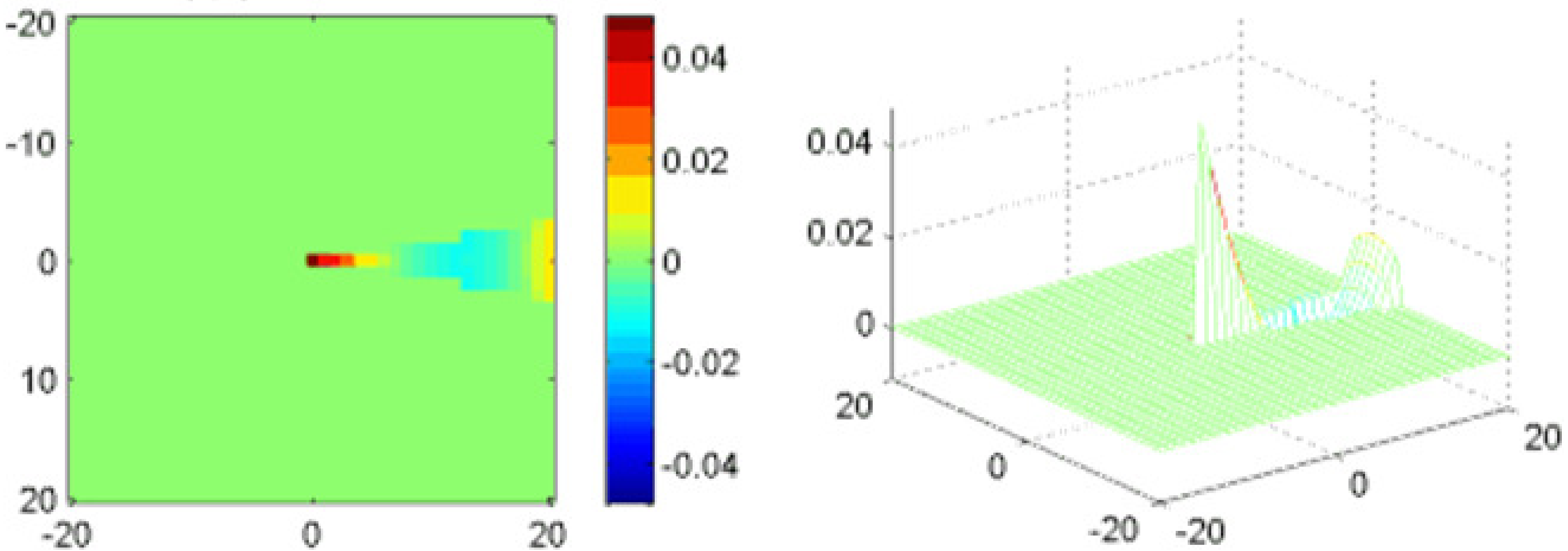
$$p_{h,m} = \operatorname{argmin}_{p \in \mathcal{P}_m} \sum_{x_s \in X} w_h(x_0 - x_s)\left(z(x_s) - p(x_s)\ \right)^2$$

$$\widehat{y_h}(x_0) = \frac{\partial^{r+c} p_{h,m}}{\partial x_1^r \partial x_2^c}(x_0)$$

- Also these estimates can be obtaine via a convolutional

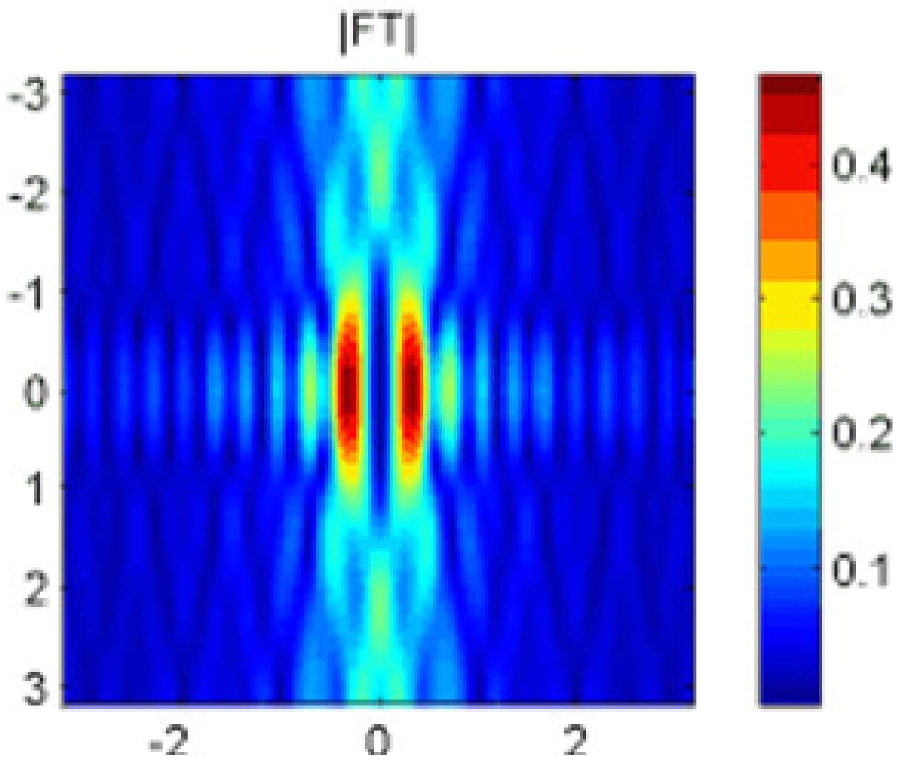
Example of LPA Anisotropic Kernels

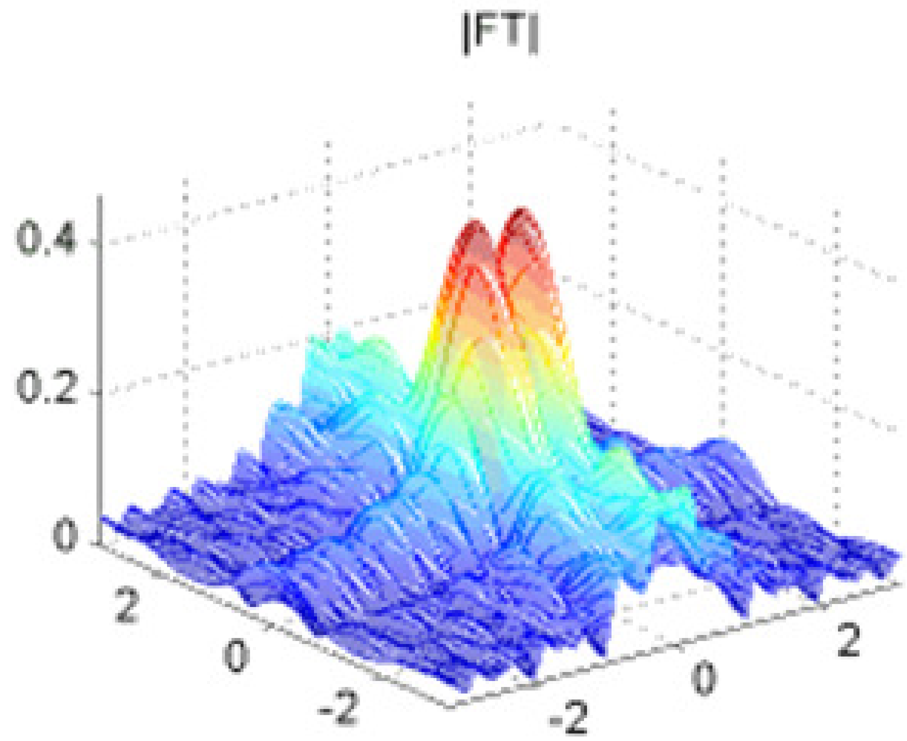
- Directional Kernel: 1° derivative along rows
  - Order along columns: 2. Order along rows: 2
  - Direction 0
  - Scale 6



Example of LPA Anisotropic Kernels

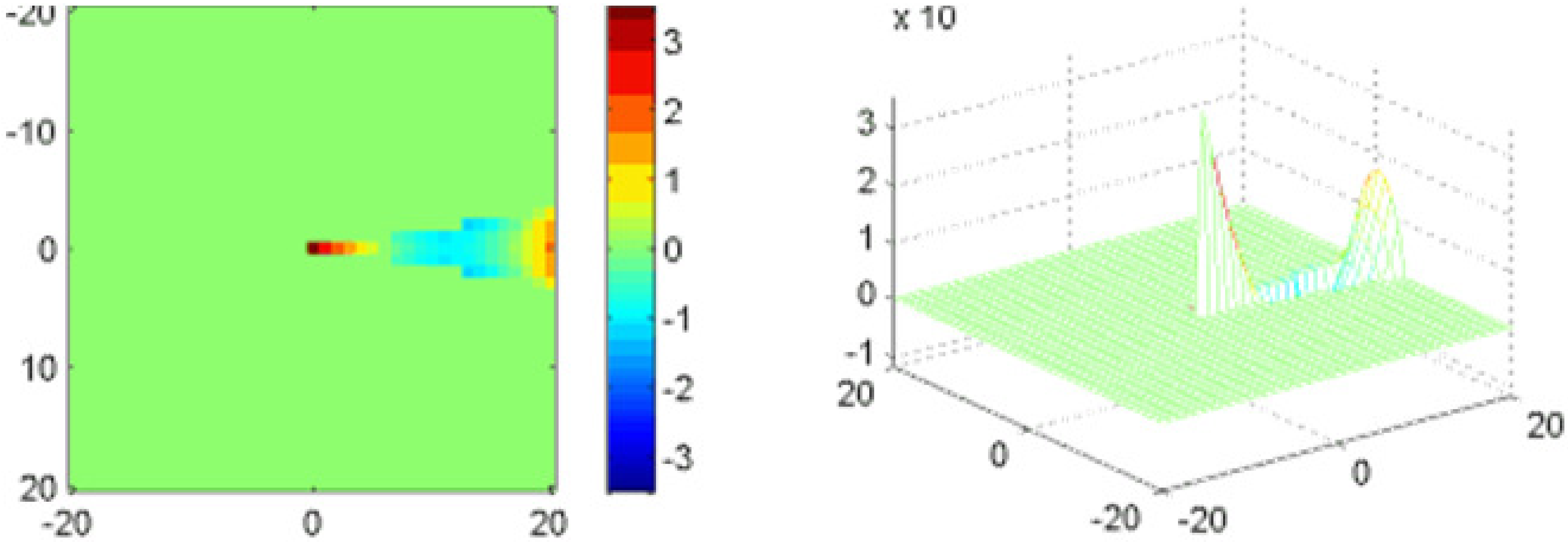
- Directional Kernel: 1° derivative along rows
  - Order along columns: 2. Order along rows: 2
  - Direction 0
  - Scale 6 Fourier Domain





Example of LPA Anisotropic Kernels

- Directional Kernel: 2° derivative along rows
  - Order along columns: 2. Order along rows: 2
  - Direction 0
  - Scale



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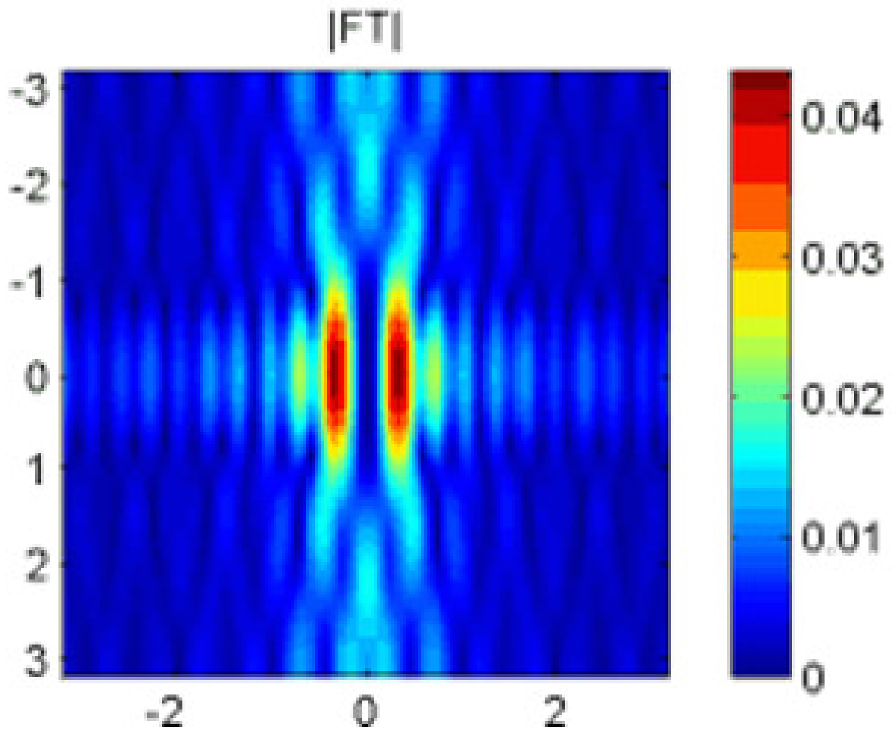
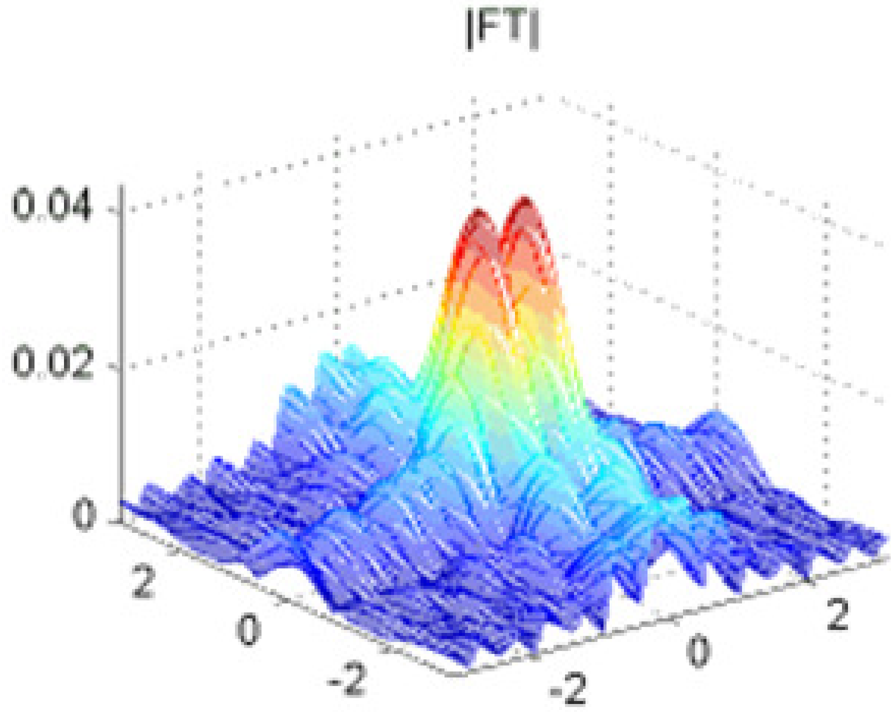




LPA Benefits

- Unlike many other transforms which start from the continuous domain and then discretized, this technique **works directly** in the **multidimensional discrete domain**;
- The LPA kernels can be designed of **any dimension**, **non-separable and anisotropic** with **arbitrary orientation**, width and length;
- Any desirable **smoothness** of the kernel can be set.
- The kernel support can be flexibly shaped to any **desirable geometry**. In this way a special design can be done for complex form objects and specific applications.
- Both **smoothing** and corresponding **differentiating** directional **kernels** can be designed.

LPA Benefits

- These kernels are by definition **asymmetric**, allowing efficient edge adaptation. Traditional symmetric or nearly-symmetric supports tend to produce either so-called ringing artifacts or oversmoothing in the vicinity of the edges.

- The Directional *LPA* allows to consider **several different problems** within a **unified framework**.

- When using LPA one implicitly assumes that the original signal is well-approximated by a polynomial in a neighborhood of each pixel: this hypothesis suits perfectly pixel-wise parallelization.

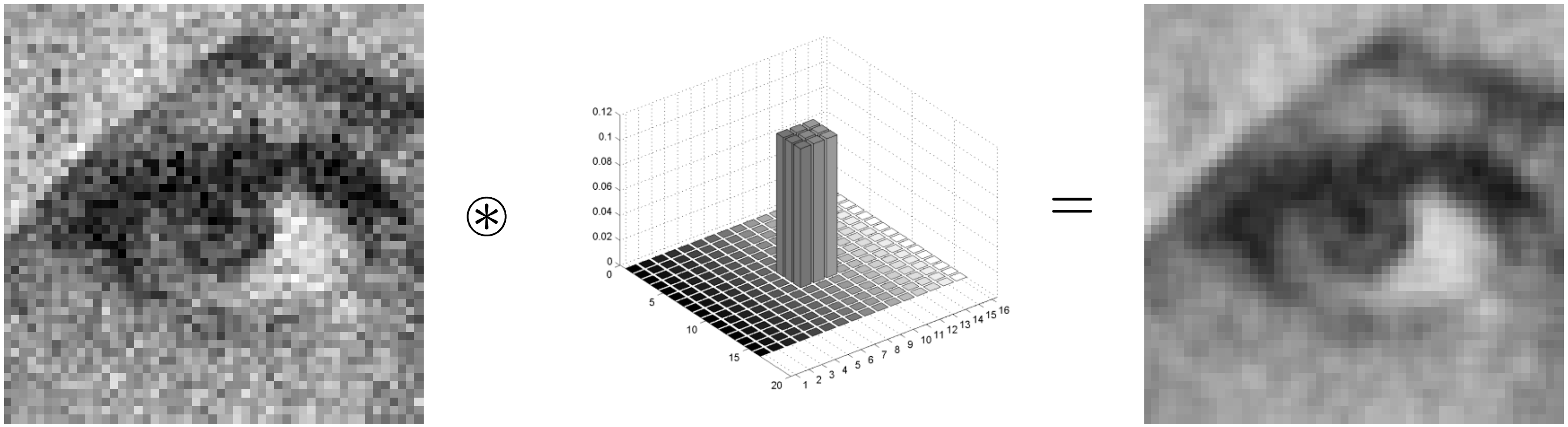
Katkovnik, V., K. Egiazarian, and J. Astola, "*Local Approximation Techniques in Signal and Image Processing*", SPIE Press, Monograph Vol. PM157, September 2006.

Foi, A., "*Anisotropic nonparametric image processing: theory, algorithms and applications,*" Ph.D. Thesis, Dip. di Matematica, Politecnico di Milano, April 2005.

Adaptiveness

- The choice of the scale parameter h is crucial as it controls the amount of smoothing in the estimation.

- Example on lena image



- Large h corresponds to less noisy output (i.e. lower variance) but typically it may result in higher bias
- Smaller h corresponds to noiser estimates (i.e. higher variance), less biased.

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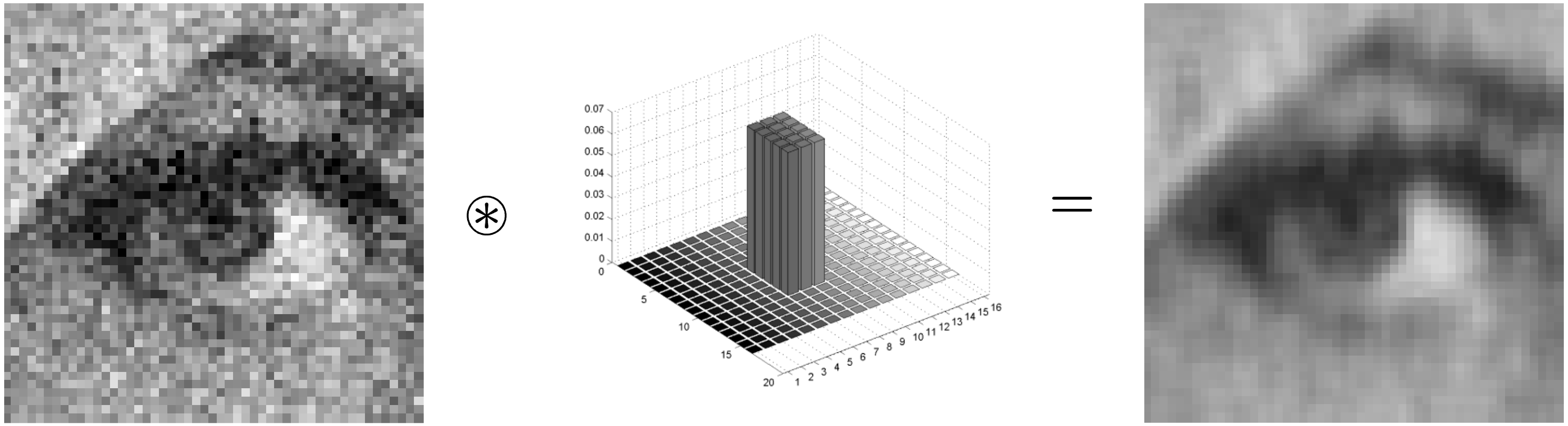

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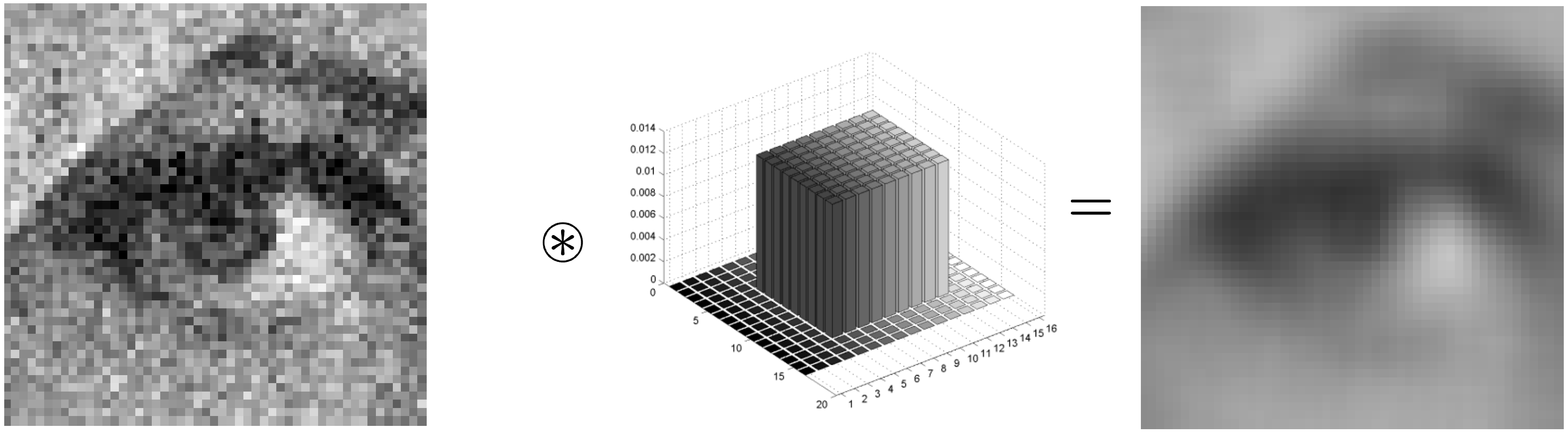

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Bias – Variance Trade Off

- Intiutively the scale parameters controls the trade off between bias and variance in the LPA estimates:
  - Bias $b_{\widehat{y}_h(x_0)} = y(x_0) - (y \circledast g_h)(x_0)$
  - Variance $\sigma^2_{\widehat{y}_h(x_0)} = (\sigma^2 \circledast g_h^2)(x_0) = \sigma^2 \|g_h\|_2^2$
- The following upper bound holds for the mean squared error

$$MSE(x) = E\left\{(y(x) - \widehat{y}_h(x_0))\right\} = b^2_{\widehat{y}_h(x_0)} + \sigma^2_{\widehat{y}_h(x_0)}$$

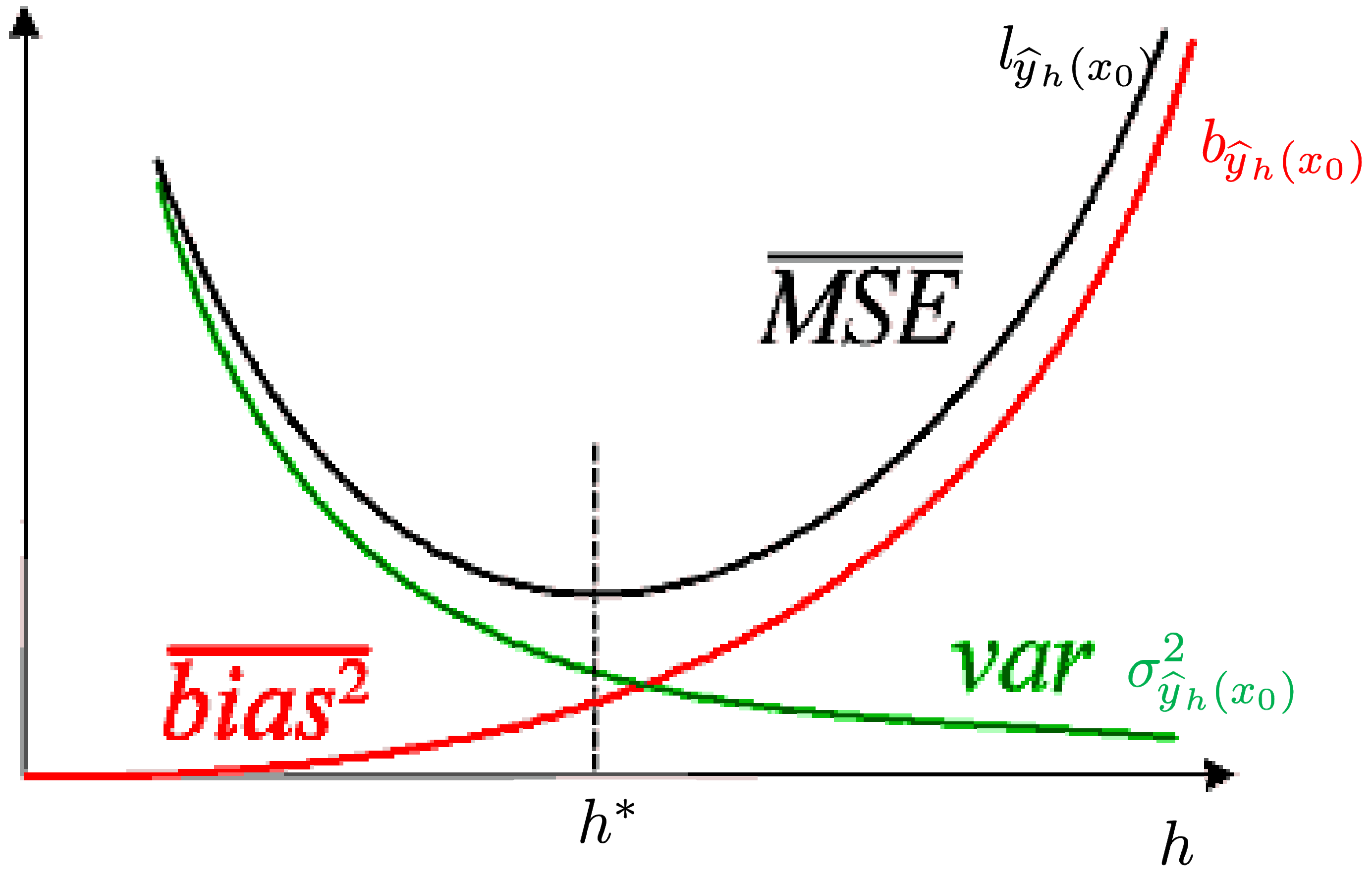
- Furthermore,the following asymptotic expressions hold for bias and the variance

$$b_{\widehat{y}_h(x_0)} \approx ch^a \qquad \sigma^2_{\widehat{y}_h(x_0)} \approx dh^{-b}$$

$$MSE(x) < l_{\widehat{y}_h(x_0)} = ch^a + dh^{-b}$$

Thus

- In *practice* this hypothesis are enough for applying the ICI rule to determine the optimal scale h^*



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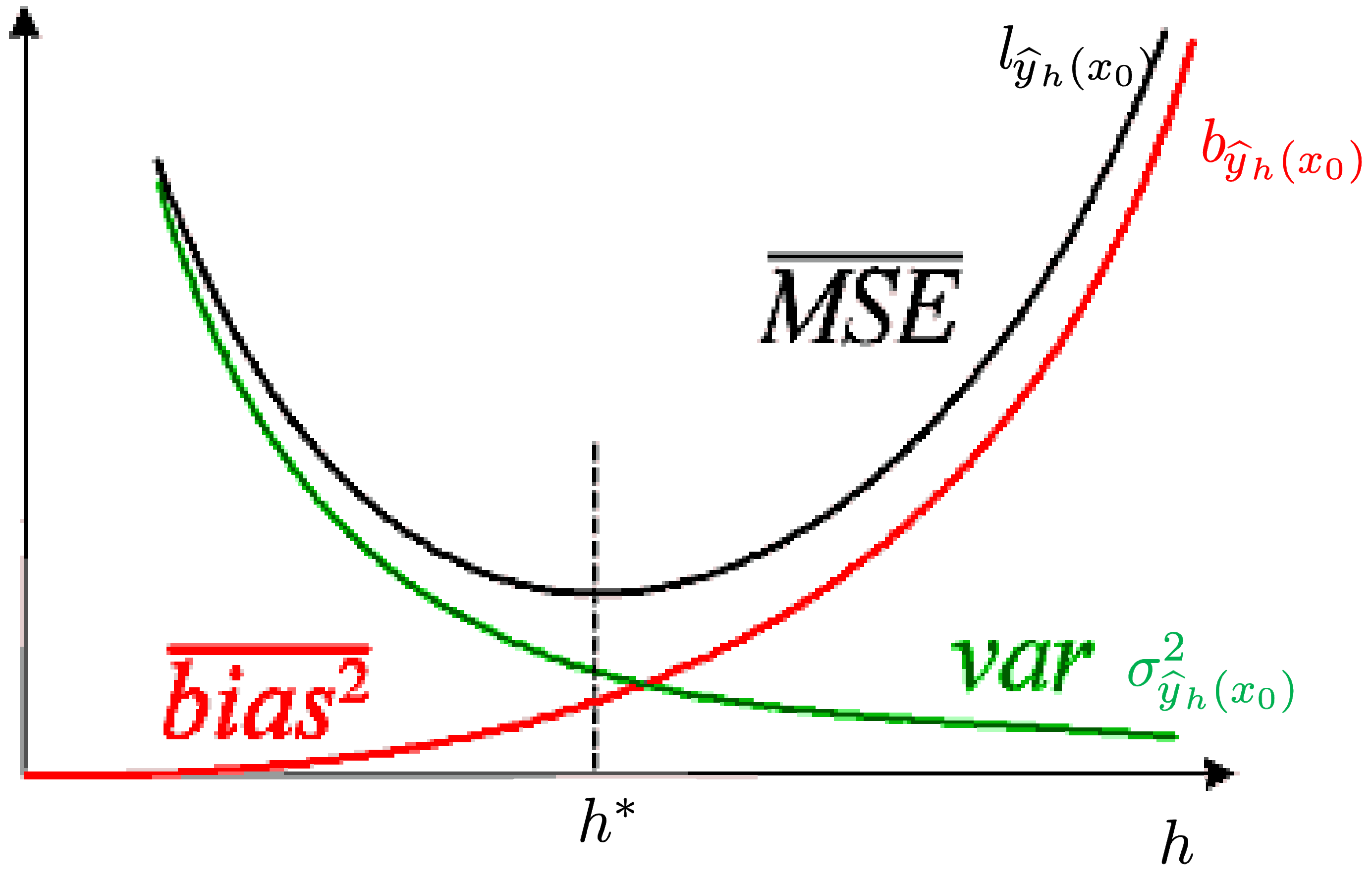


- The ICI rule is used to determine kernel scale h^+ that approximates h^*

The ICI rule

- Consider a fixed kernel direction d :
- For each pixel x , the estimates $\widehat{y}_{h,d}(x)$ are computed on a set of increasing scales $h \in H = \{h_j\}_{j=1}^{J}$
- For each estimate we can build a confidence interval as follows

$$\mathcal{D}_{h,d}(x) = [\widehat{y}_{h,d}(x) - \Gamma\sigma_{h,d},\ y\hat{_{h,d}}(x) + \Gamma\sigma_{h,d}]$$

where $\Gamma > 0$ is a tuining parameter

- The ICI rule yields a pointwise adaptive estimate $\widehat{y}_{h^+,d}(x)$ such that

$h^+ \approx h^*$ $h \in H$ in a sense that $\widehat{y}_{h^+,d}(x) \approx \widehat{y}_{h^*,d}(x)$

Goldenshluger, A., and A. Nemirovski, *"On spatial adaptive estimation of nonparametric regression"*, Math. Meth. Statistics, vol. 6, pp. 135-170, 1997

The ICI rule

- The ICI rule can be state as follows:

Consider the intersection of confidence intervals

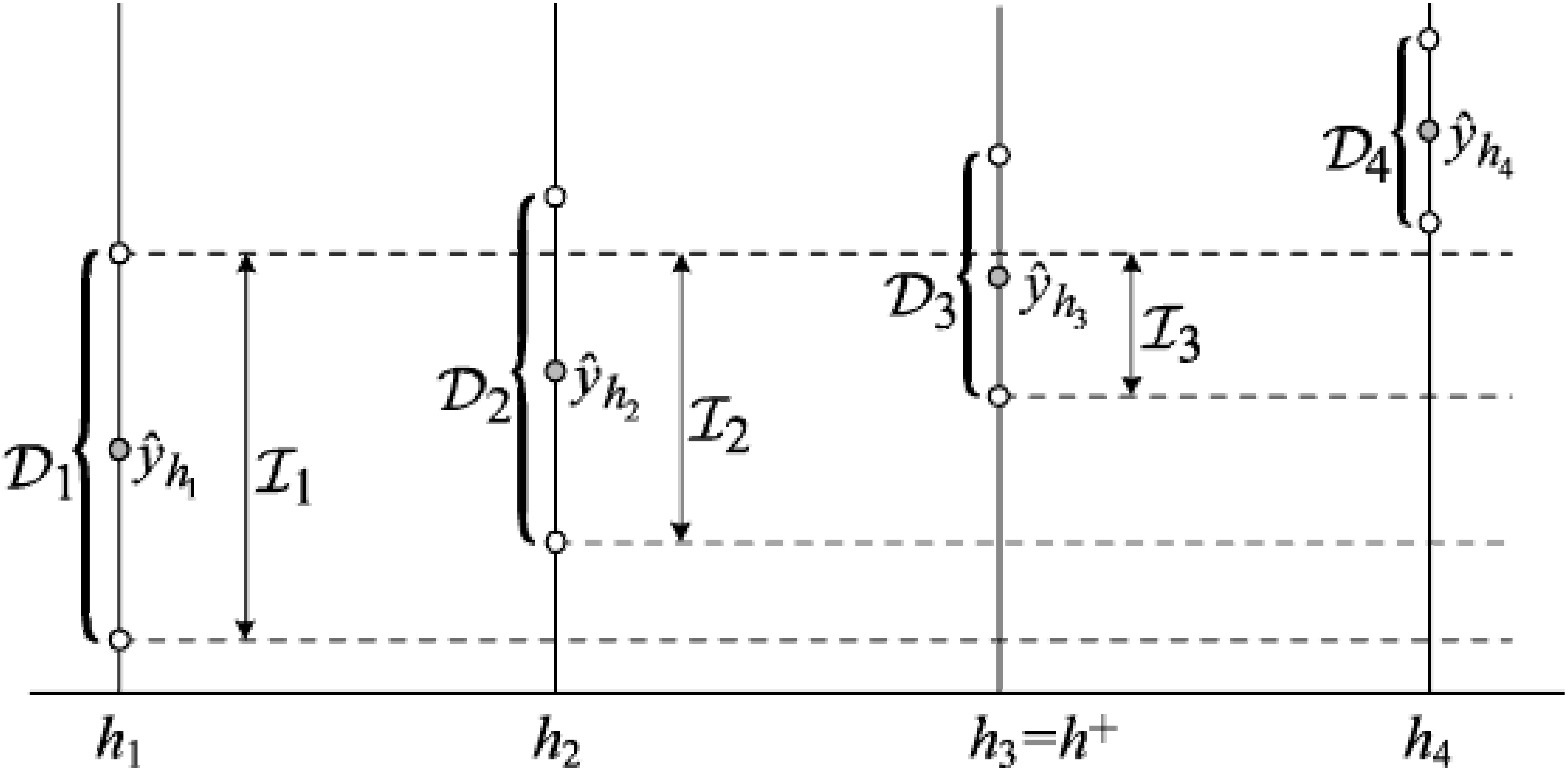
$$\mathcal{I}_{h,d}(x) = \bigcap_{h_j \leq h} \mathcal{D}_{h_j,d}(x)$$

where $\mathcal{D}_{h,d}(x) = [\widehat{y}_{h,d}(x) - \Gamma\sigma_{h,d},\ \hat{y_{h,d}}(x) + \Gamma\sigma_{h,d}],\ \Gamma > 0$.

Then let j^+ *be the largest of the scale indexes for which* $\mathcal{I}_{j^+,d} \neq \emptyset$ *and* $\mathcal{I}_{j^+ +1,d} = \emptyset$ *: then,*h^+ *is defined as* $h^+ = h_{j^+}$ *and the adaptive estimate is* $\widehat{y}_{h^+,d}(x)$ *.*

Goldenshluger, A., and A. Nemirovski, ***"On spatial adaptive estimation of nonparametric regression"***, Math. Meth. Statistics, vol. 6, pp. 135-170, 1997

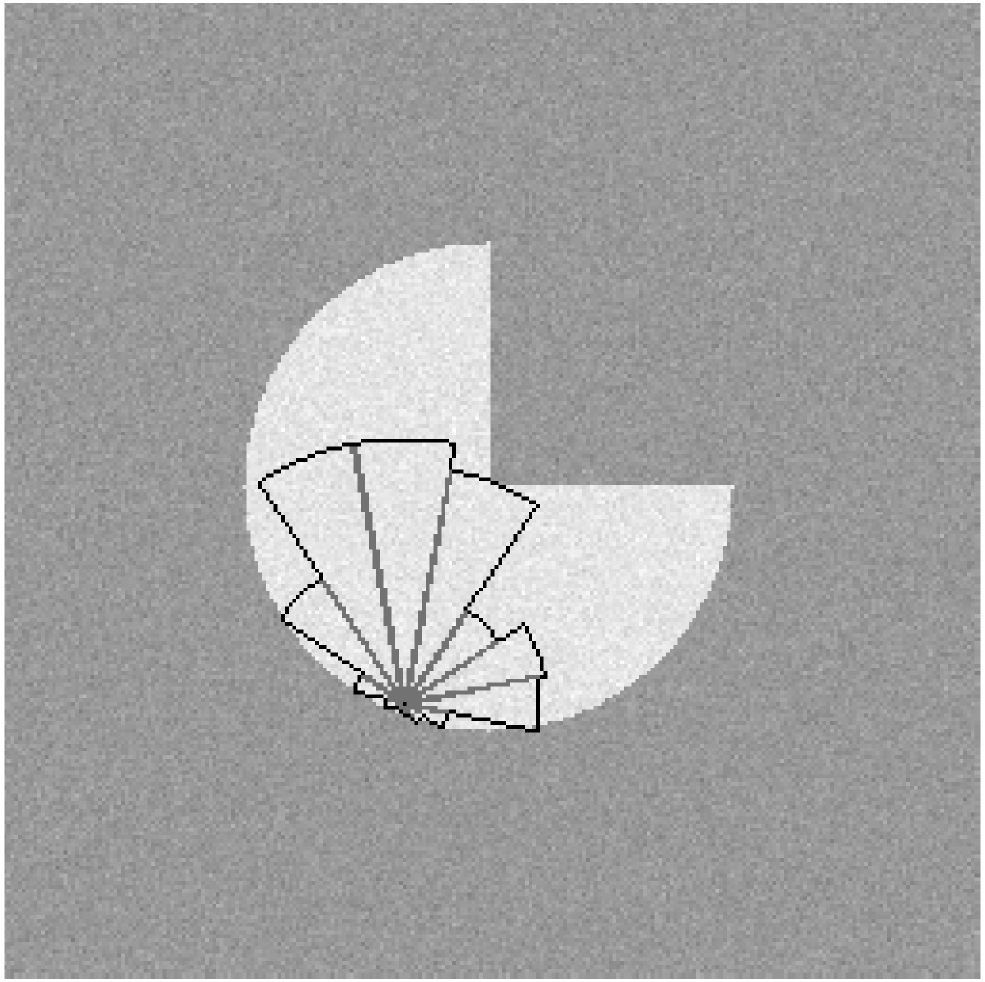
The ICI rule



Goldenshluger, A., and A. Nemirovski, *"On spatial adaptive estimation of nonparametric regression"*, Math. Meth. Statistics, vol. 6, pp. 135-170, 1997

Examples of adaptively selected neighborhoods

- Adaptively selected neighborhoods selected using the LPA-ICI rule

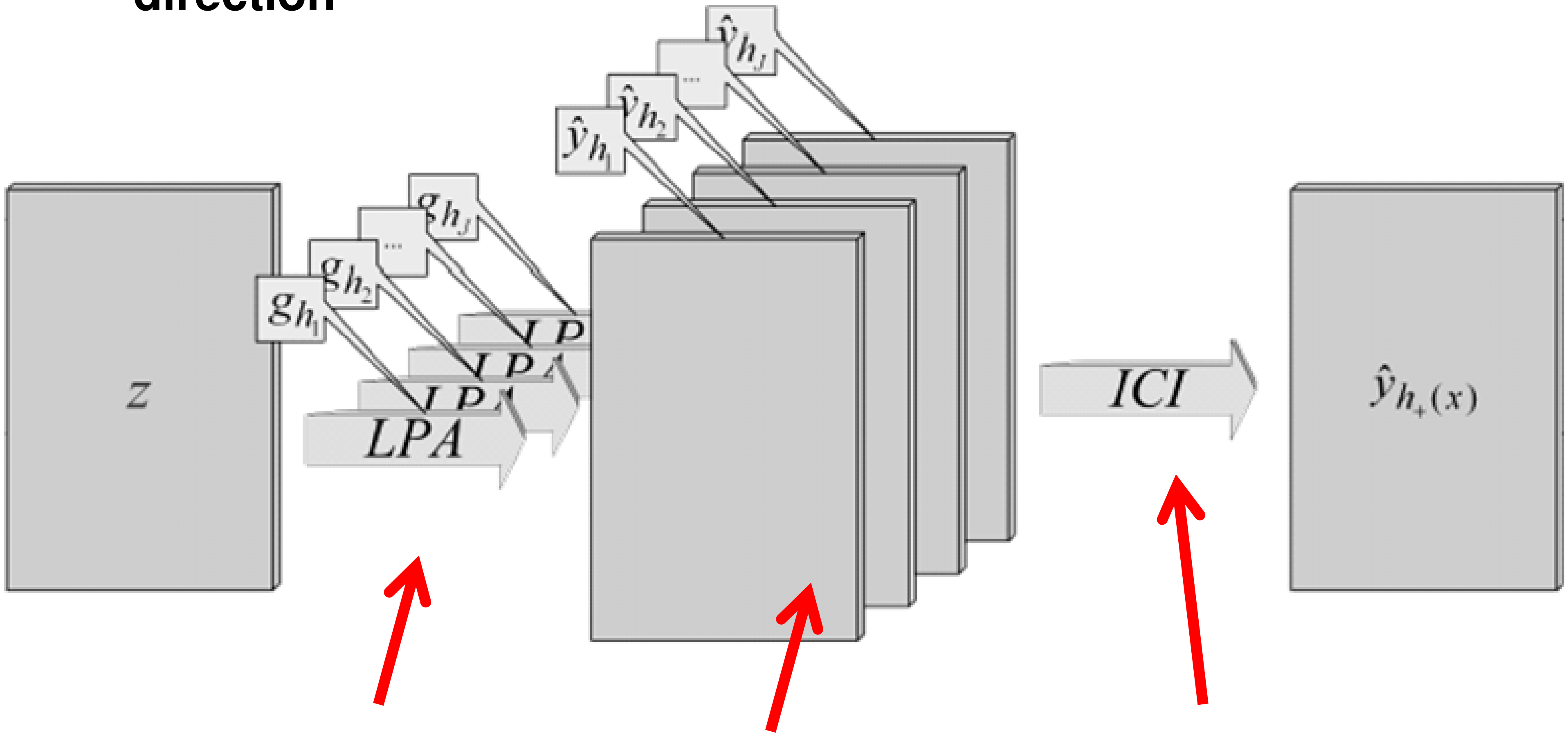


LPA-ICI Denoising Algorithm Details

1. Compute the ICI-selected scales along each direction
2. ICI-selected scales filtering and Update Directional Estimates
3. Fusing of directional estimates

ICI scale selection - Matlab implementation-

- It is typically faster to perform 2D-convolution **- given a kernel direction -**

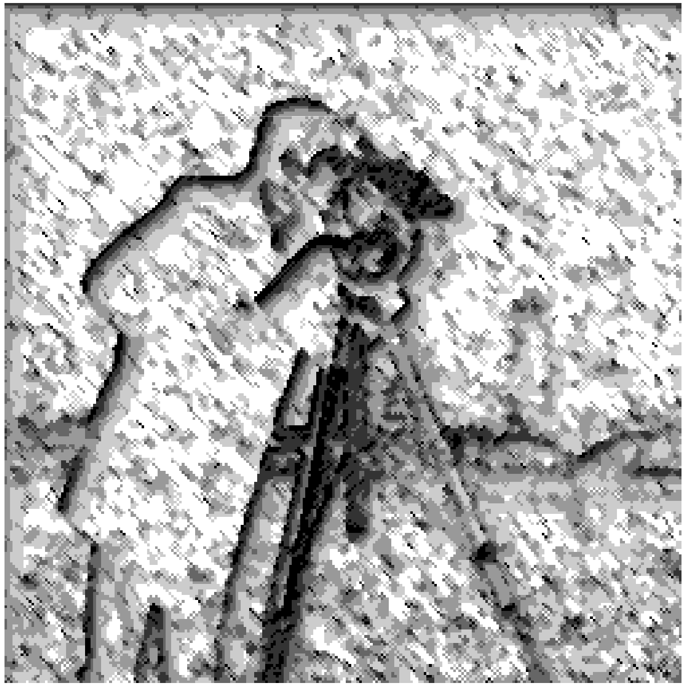


Compute *H* image convolutions, one per each kernel length

Compute the confidence intervals

For each pixel, compute the Intersection of Confidence Intervals

Example ICI selected Scales and Directional Estimates



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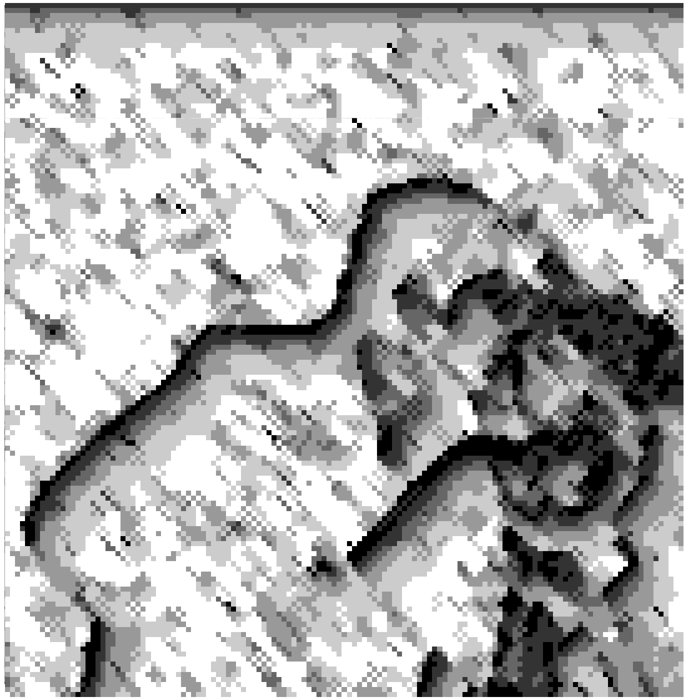



Step 2: ICI-selected scales filtering

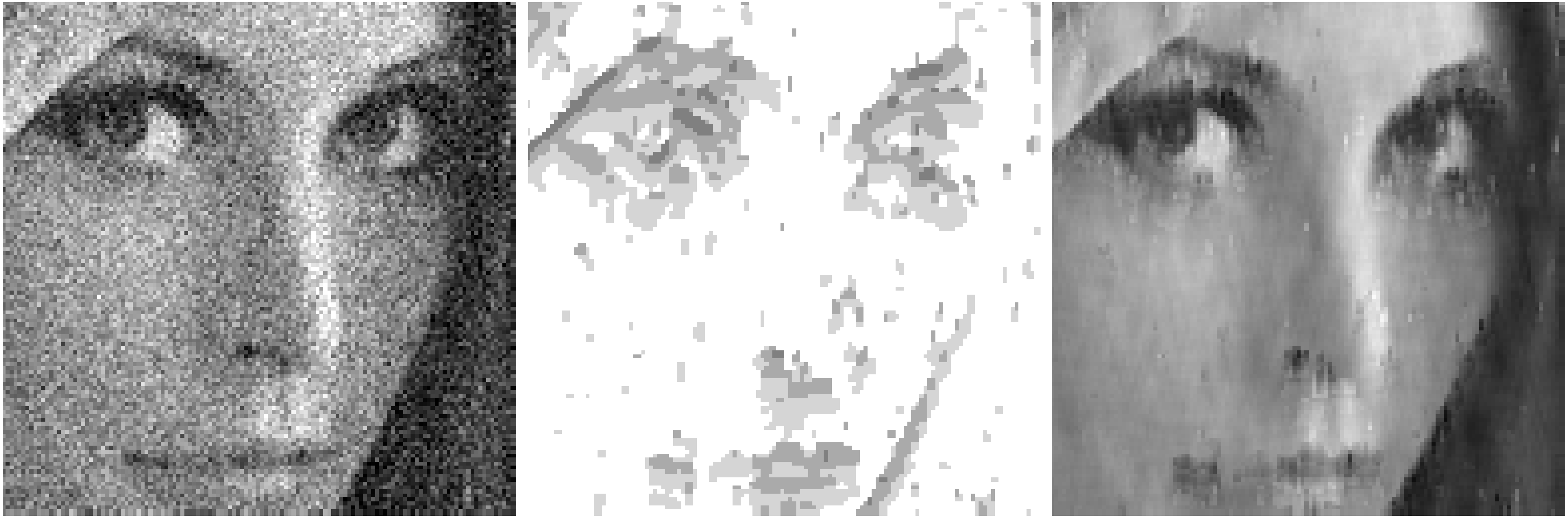
- Use a **median filter** or a **Weight Order Statistics** filter on the **ICI selected scales**
  - This operation is typically performed separately for the scales selected along each direction
  - When using WOS, these are directed “orthogonally” to anisotropic LPA-Kernels
- The “old” directional estimates are replaced by the LPA estimates and the standard deviations corresponding to the scales filtered.
- In such a way we **remove isolated** pixels where the **ICI selected “the wrong scale”**

Example ICI selected scales



Example ICI selected scales

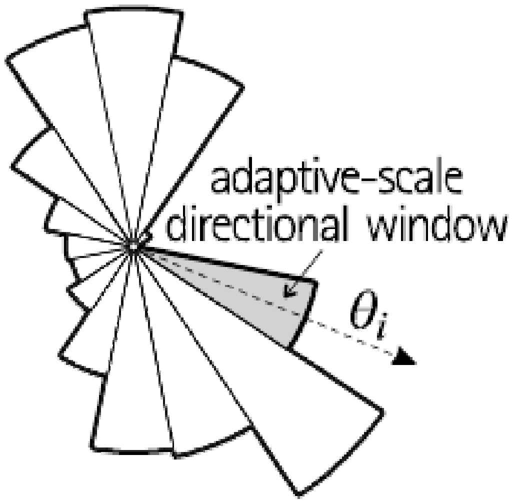
Example ICI selected scales

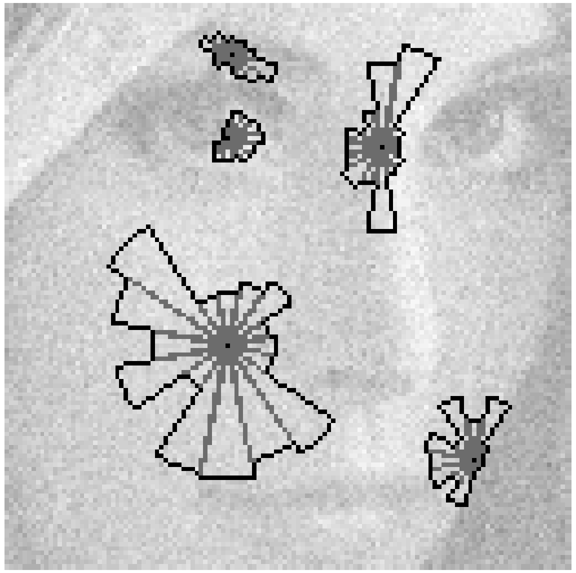


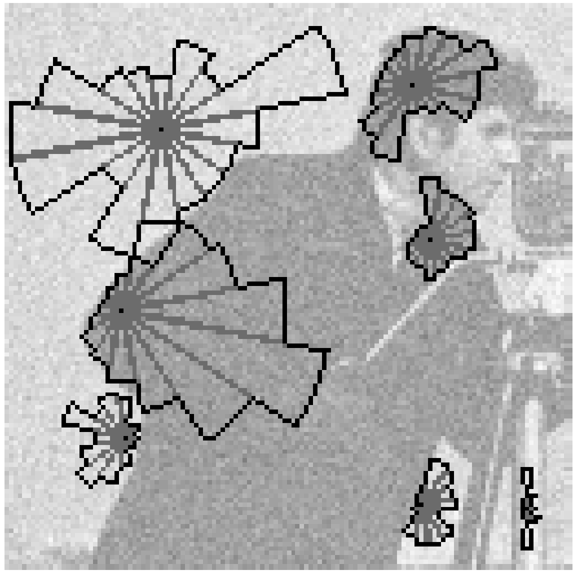
Step 3: Fusing of directional estimates

- For each pixel
  - Fuse the directional estimates with a convex linear combination the weights are determined by the inverse of the directional estimates variance

$$\hat{y}(x) = \sum_d \frac{\widehat{y}_{d,h^+}(x)}{\sigma^2_{d,h^+}(x)} \; / \; \sum_d \sigma^{-2}_{d,h^+}(x)$$







  - In such a way, larger weights are assigned to the less noisy estimates

Example of Final Estimate

- Directional Estimates

Fused Estimate

Denoising Performance

- Details: original image (top left), anisotropic *LPA-ICI*, *ISNR*=8.2dB (top right), TI wavelets (DB4), *ISNR*=7.4dB (bottom left), TI wavelets (Haar), *ISNR*=7.8dB (bottom right).

Implementation Details

1. Compute the ICI-selected scales along each direction
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These instructions will be performed on a Cuda Kernel, and parallelized for each pixel.

Step 1 + 3 : Compute ICI-selected scales

- For each pixel
  - For each LPA kernel direction *d*
  - *// initialize the ICI selected scale* $h_d^+(x) = J$
    - For each scale kernel $h \in H$

      // Compute the LPA estimate at x and its standard deviation

      $\widehat{y}_{h,d}(x) = (z \circledast k_{h,d})(x) \qquad \sigma_{h,d} = \|k_{h,d}\|_2^2 \sigma$
    - *// Determine confidence interval*

      $\mathcal{D}_{h,d}(x) = [\widehat{y}_{h,d}(x) - \Gamma\sigma_{h,d},\ \hat{y_{h,d}}(x) + \Gamma\sigma_{h,d}]$
    - *// Determine Intersection of Confidence Intervals*

      $$\mathcal{I}_{h,d}(x) = \bigcap_{h_j \leq h} \mathcal{D}_{h_j,d}(x)$$
    - *If* $\mathcal{I}_{h,d} = \emptyset$
      - *// The ICI selected scale is h-1*
      - $h_d^+(x) = h - 1$
      - *Break*
  - *// fuse the directional estimates*

Implementation Details

- The Γ parameter determine the width of Confidence Intervals:
  - Larger Γ tend to select larger scales
  - Smaller Γ tend to select smaller scales

- The convolution has to be computed also at image boundaries.
  - An efficient and practical solution in this case consist of padding the original image with a value much smaller than 0 (e.g. -10000)

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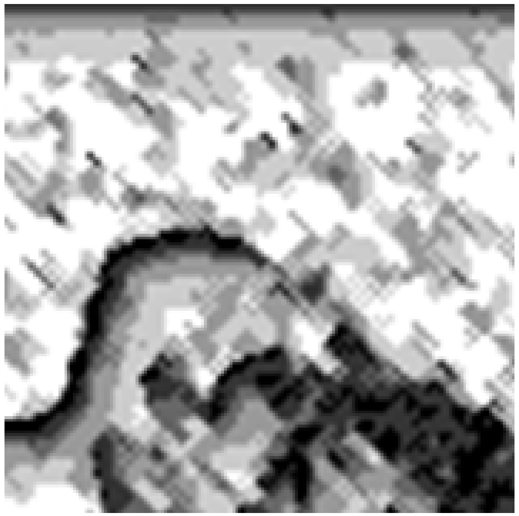
Add PADDING area

Compute convolution against the padded image

Compute scales on PADDING free image



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Donoho, D.L., and I.M. Johnstone, *"Ideal spatial adaptation via wavelet shrinkage"*, Biometrika, n. 81, pp. 425-455, 1994

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- As an error metric you can use RMSE

$$RMSE(\widehat{y}) = \sqrt{\frac{\sum_x \left(\widehat{y}(x) - y(x)\right)^2}{\#X}}$$

Other Applications

- Deblurring

Other Applications

- Deblurring

Method Experiment	1	2	3	4
LPA-ICI directional	8.23	7.78	6.04	3.76
GEM (Dias)	8.10	7.47	5.17	–
EM (Figueiredo and Nowak)	7.59	6.93	4.88	2.94
ForWaRD (Neelamani et al.)	7.30	6.75	5.07	2.98

ISNR

Other Applications

- Inverse Halftoning

Other Applications

- Inverse Halftoning

original grayscale image (top left), binary Floyd-Steinberg halftone (top right), anisotropic *LPA-ICI* estimate, *PSNR*=32.4dB (bottom left) and wavelet-based *WinHD* estimate (Rice Univ.), *PSNR*=32.1dB (bottom right)