

# Anomaly Detection in Images

Giacomo Boracchi,

DEIB, Politecnico di Milano

Università Bicocca, February 26<sup>th</sup> 2025

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<https://boracchi.faculty.polimi.it/>



**POLITECNICO**  
MILANO 1863

# Giacomo Boracchi



Mathematician (Università Statale degli Studi di Milano 2004),  
PhD in Information Technology (DEIB, Politecnico di Milano 2008)  
Associate Professor since 2019 at DEIB (Computer Science), Polimi

Research Interests are mathematical and statistical methods for:

- Image / Signal analysis and processing
- Unsupervised learning, change / anomaly detection

Major Courses:

- Mathematical Models and Methods for Image Processing (MSc, Polimi)
- Artificial Neural Networks and Deep Learning (MSc Polimi, Bocconi)
- Advanced Deep Learning Models And Methods (PhD, Polimi)
- Computer Vision (MSc Bocconi, USI 2020)

# The Team

We are 3 faculties, 10 PhD students, 1 Research Assistant... and 20+ MSc students!



*Giacomo Boracchi*



*Luca Magri*



*Federica Arrigoni*



*Riccardo  
Margheritti*



*Michele Craighero*



*Edoardo Peretti*



*Andrea Diecidue*



*Roberto Basla*



*Rakshith  
Madhavan*



*Andrea Ferraris*



*Diego Martin*



*Luca Alessandrini*



*Andrea  
Marelli*

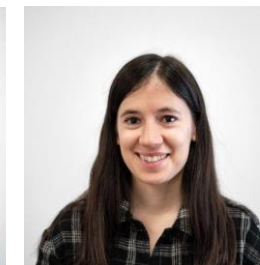


*Carlo Sgaravatti*



# Computer Vision Research

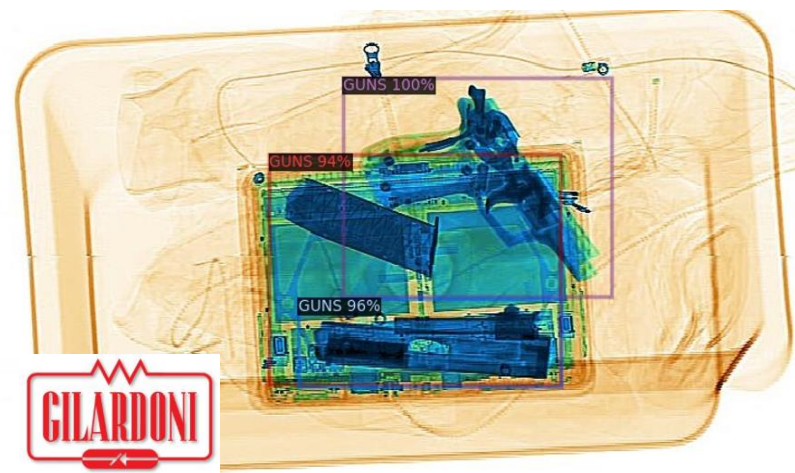
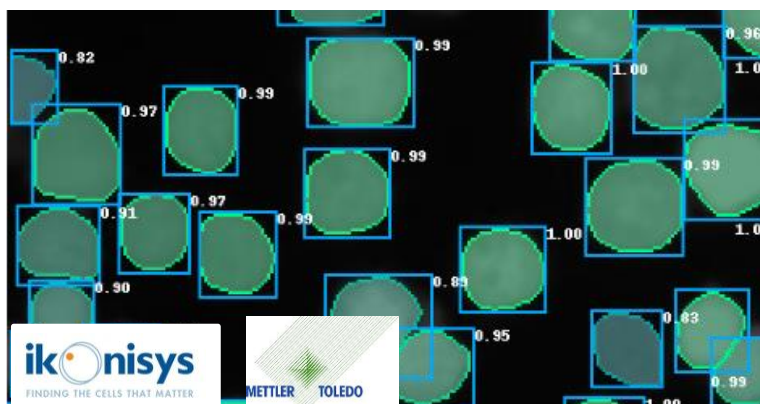
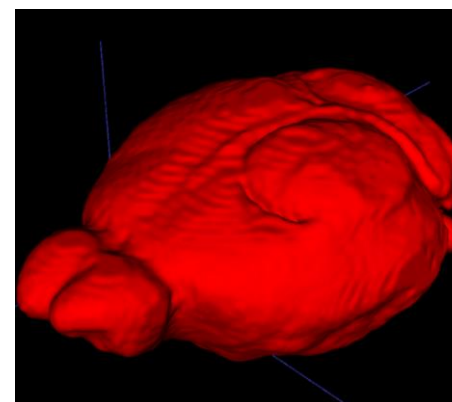
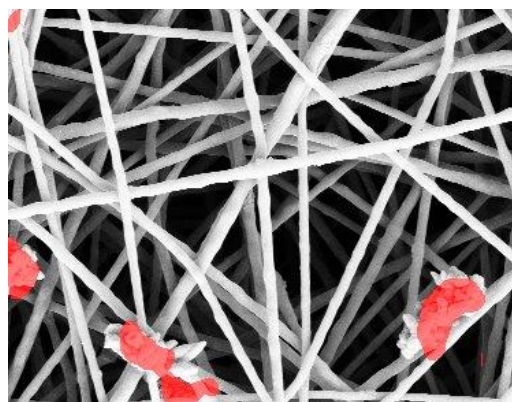
Giacomo Boracchi, Luca Magri, Federica Arrigoni



3 faculties  
10 PhD students  
1 Research  
Assistants

## Research themes and relevant projects:

- **Deep Learning for Visual Recognition:** Object Detection and Segmentation (CCD, SPAD, X-ray, 2D/3D Medical, Aerial Images), Anomaly Detection (images, signals), Learning under limited supervision.



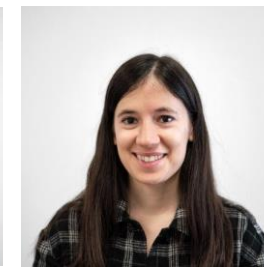
PERIVALLON

with Prof. Piero Fraternali



# Computer Vision Research

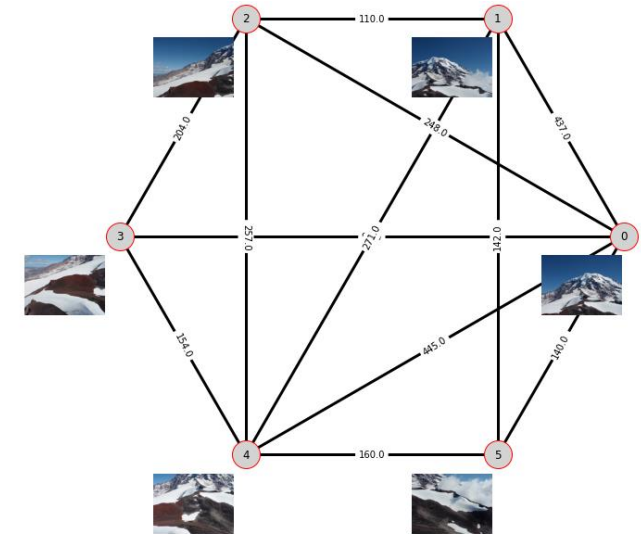
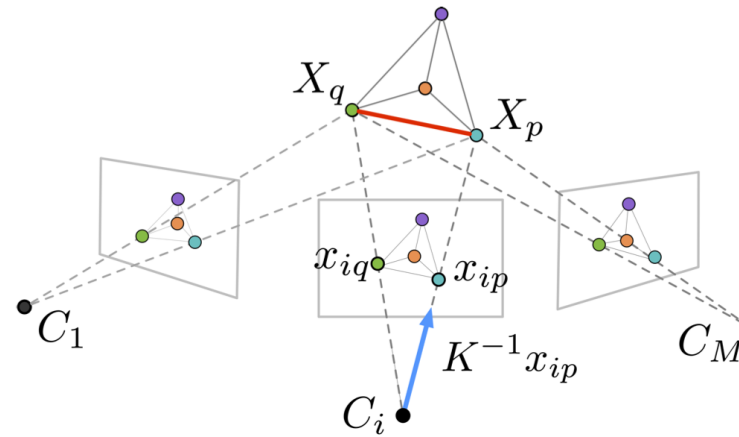
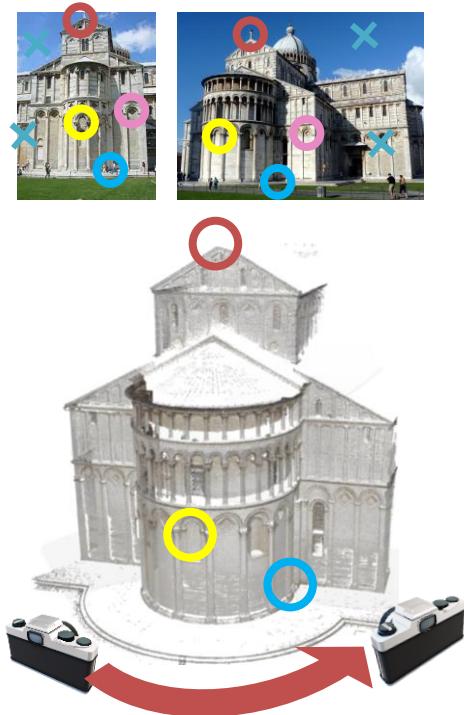
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# Computer Vision Research

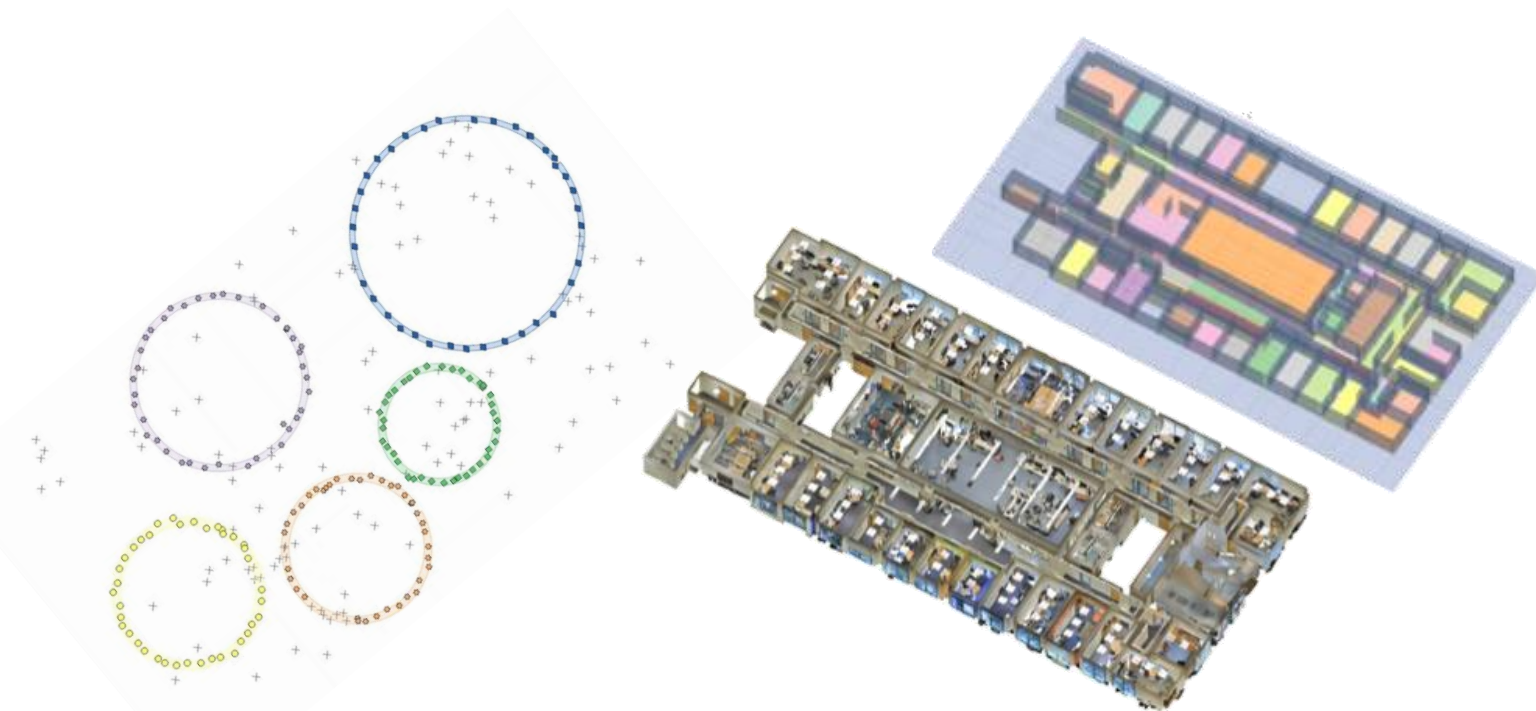
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# Computer Vision Research

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Assistants

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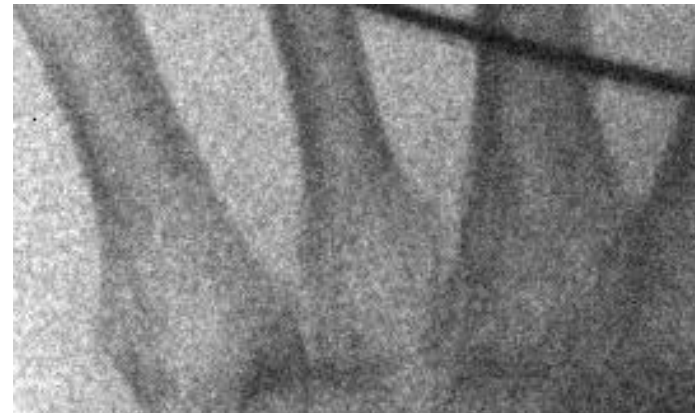
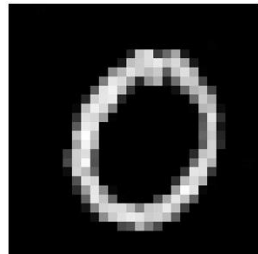
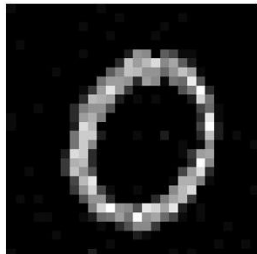
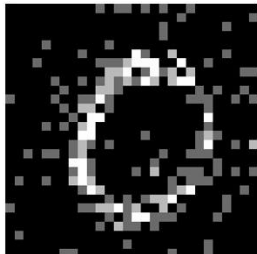
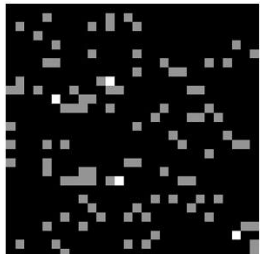
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5 mlux:  
mean count 0.13

40 mlux:  
mean count 0.37

320 mlux:  
mean count 2.28

2560 mlux:  
mean count 17.44





# Computer Vision Research

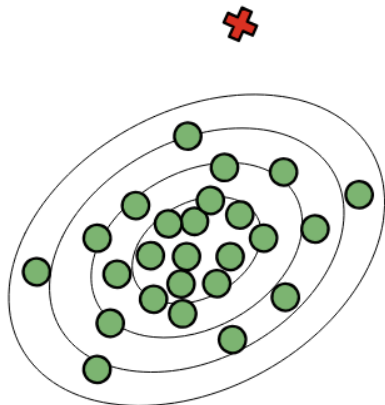
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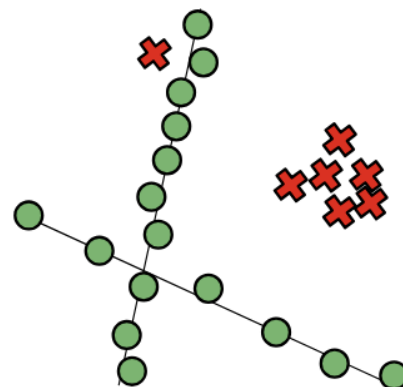
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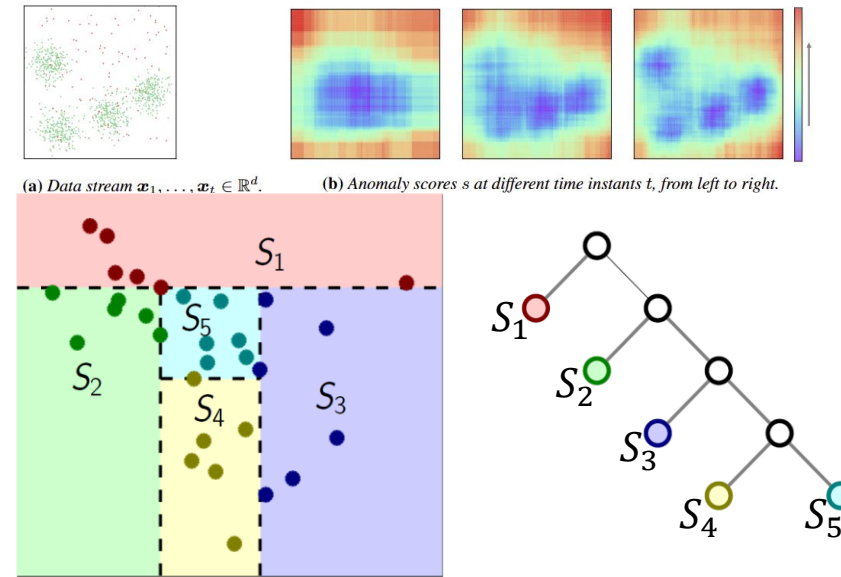
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- **Image/Signal Processing and Analysis:** Image Enhancement and Restoration (X-ray, CCD, SPAD), Change and Outlier Detection (Optical fiber signals)
- **Change and Anomaly Detection:** Design of change-detection tests for high-dimensional data (QuantTree), Sequential Monitoring, Datastream Mining, Anomaly Detection (tree based methods).



(a) Density.



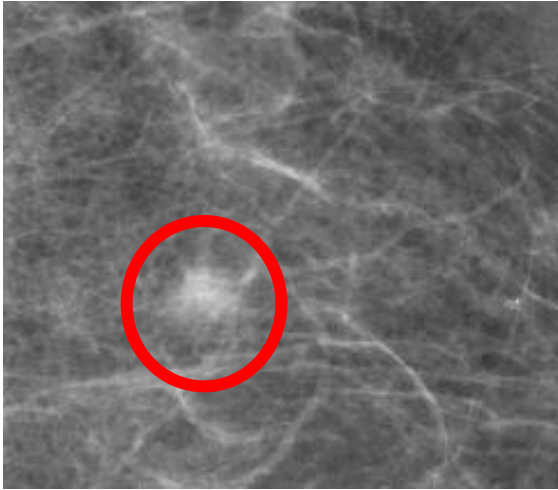
(b) Structure.



# Anomalies

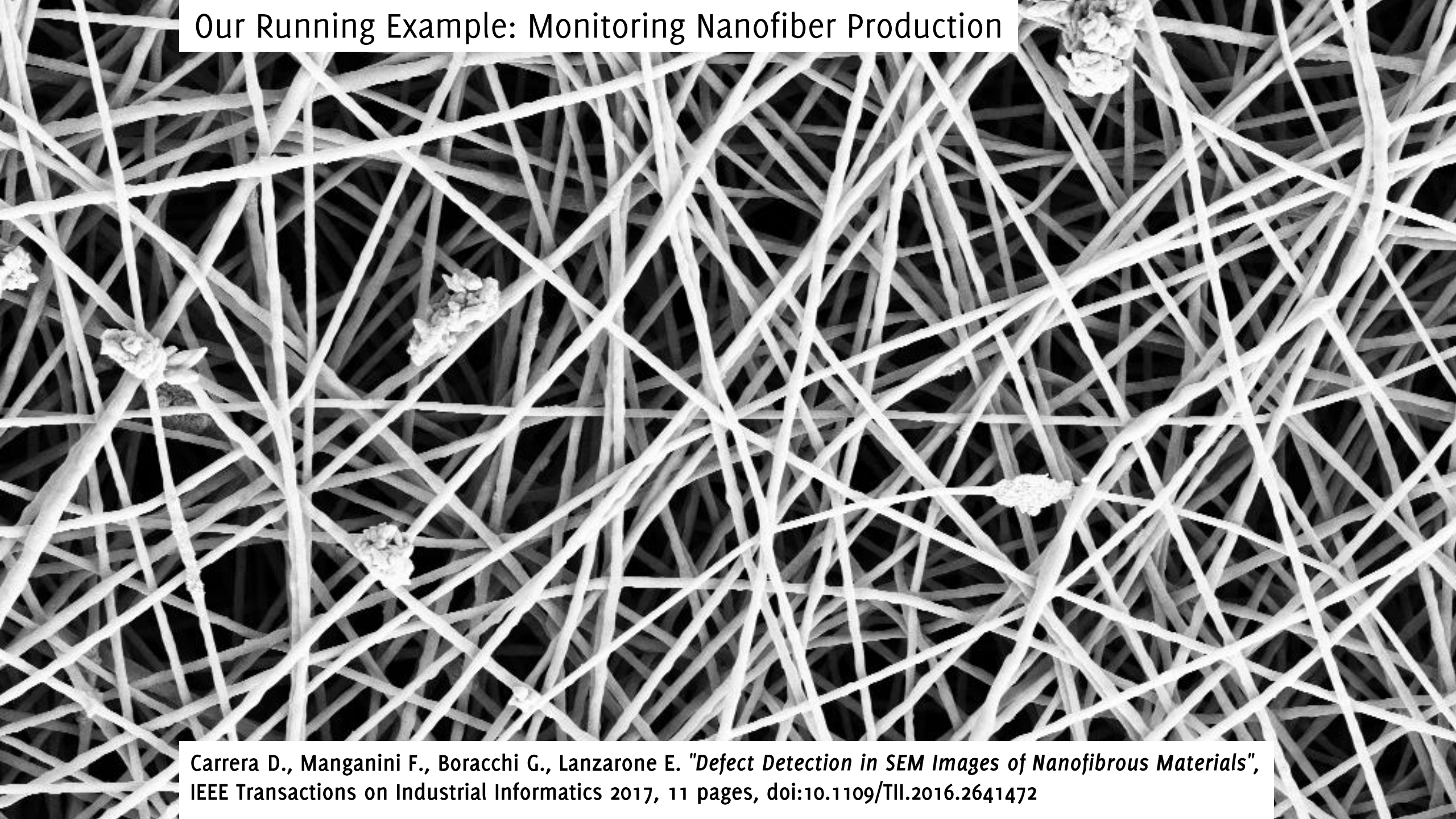
# Anomaly detection in health

**Mammograms**





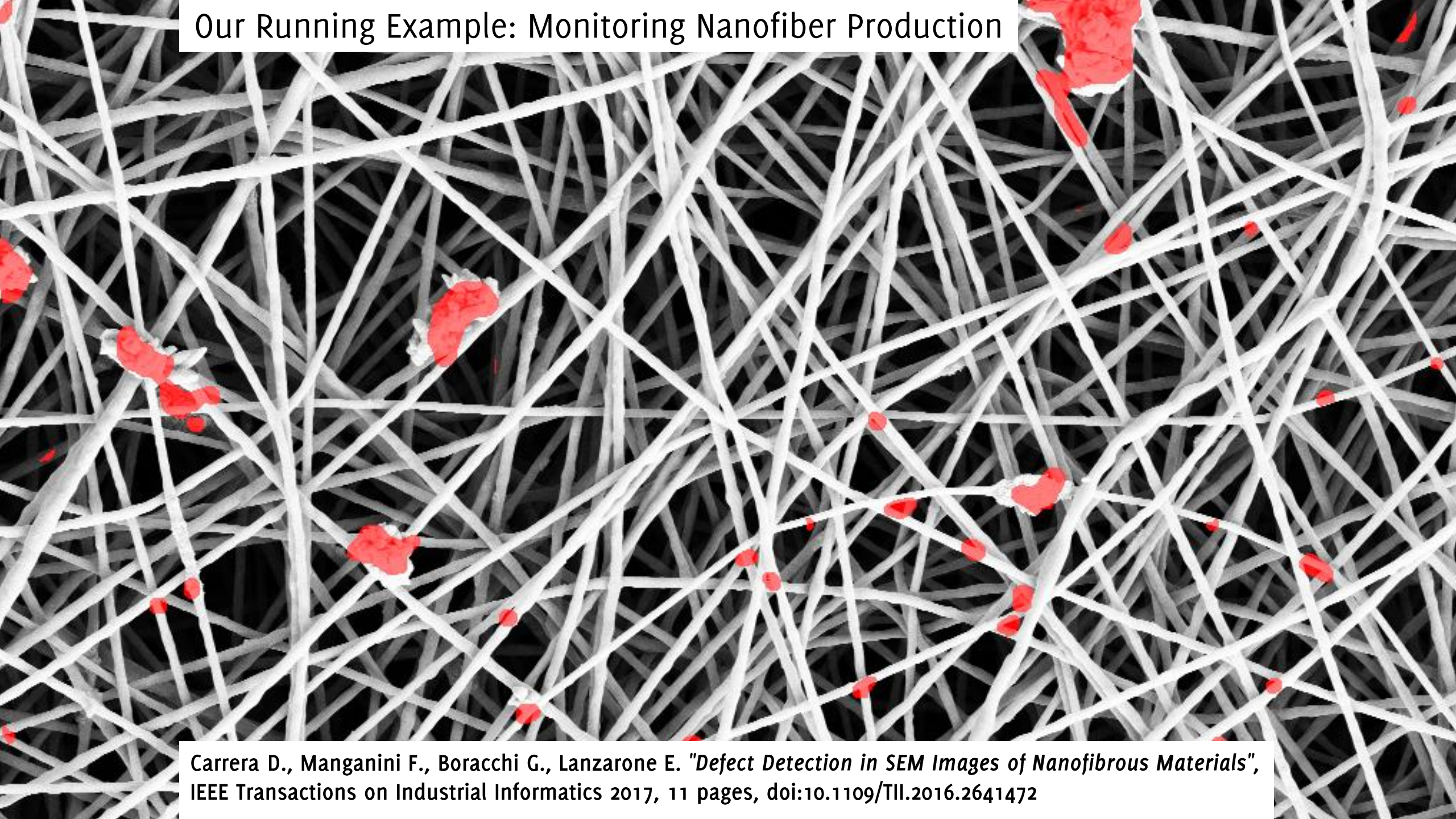
## Our Running Example: Monitoring Nanofiber Production



Carrera D., Manganini F., Boracchi G., Lanzarone E. *"Defect Detection in SEM Images of Nanofibrous Materials"*, IEEE Transactions on Industrial Informatics 2017, 11 pages, doi:10.1109/TII.2016.2641472



## Our Running Example: Monitoring Nanofiber Production

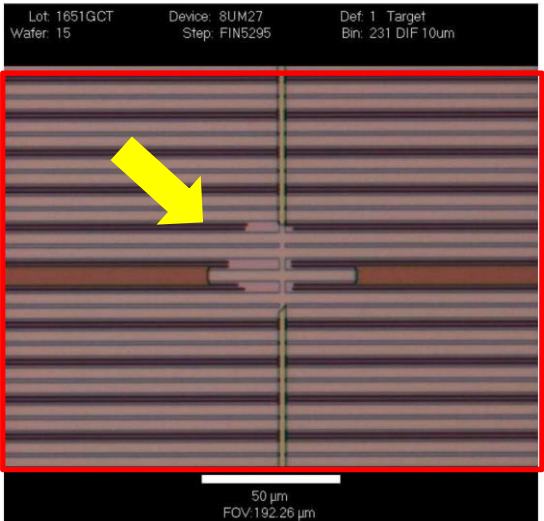
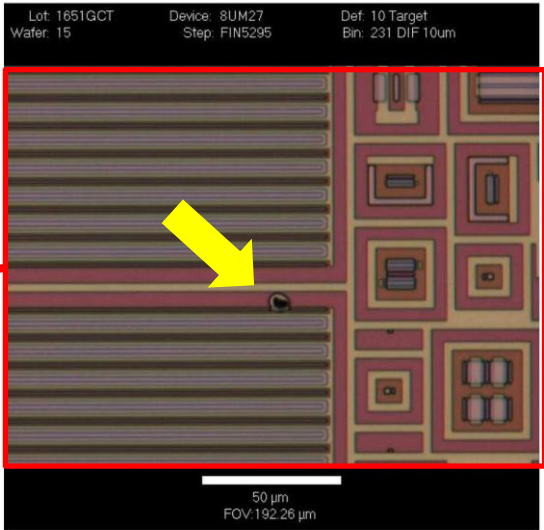
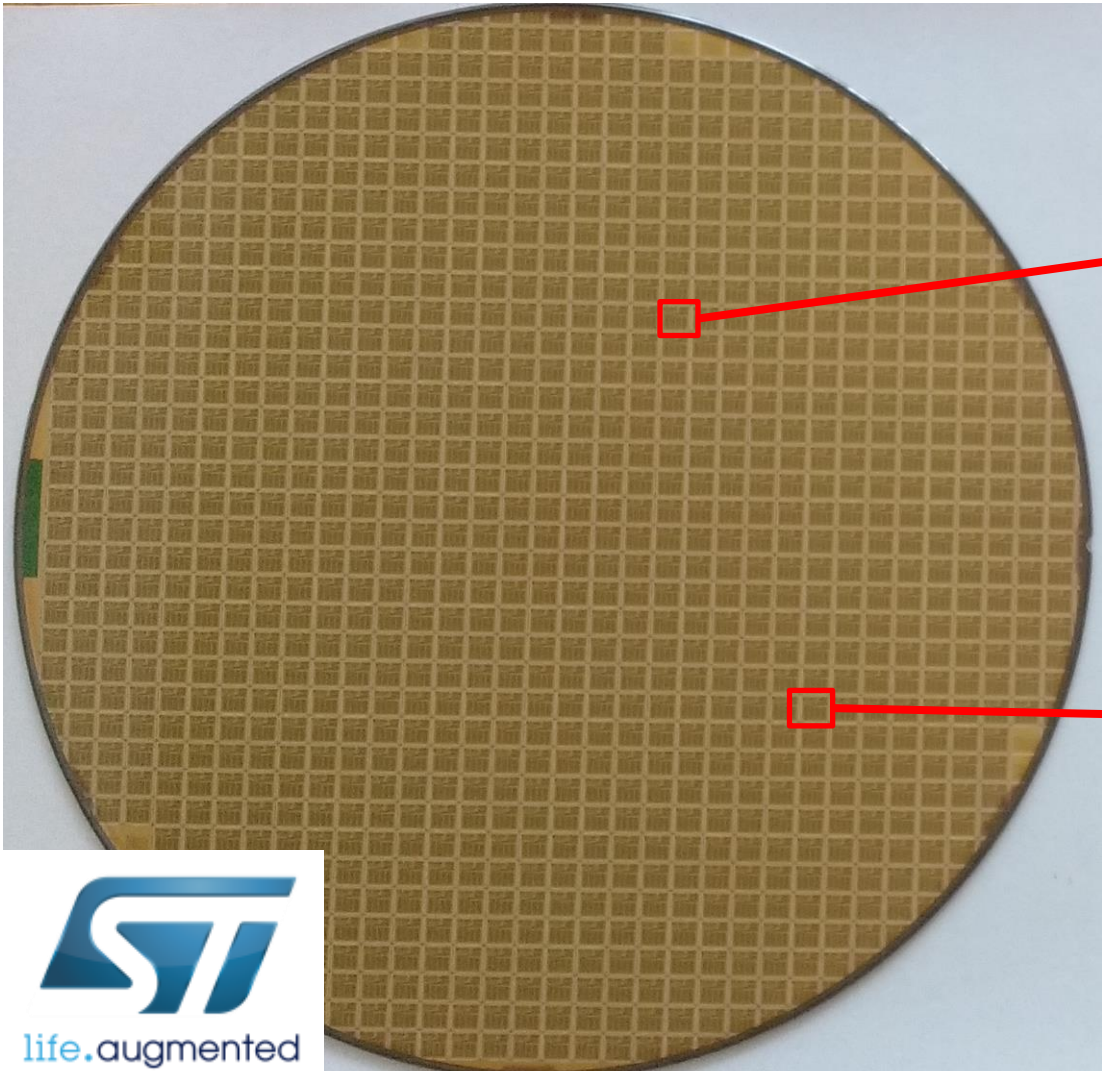


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# SYLICON WAFER MANUFACTURING

Defects detected as anomalies in microscope images

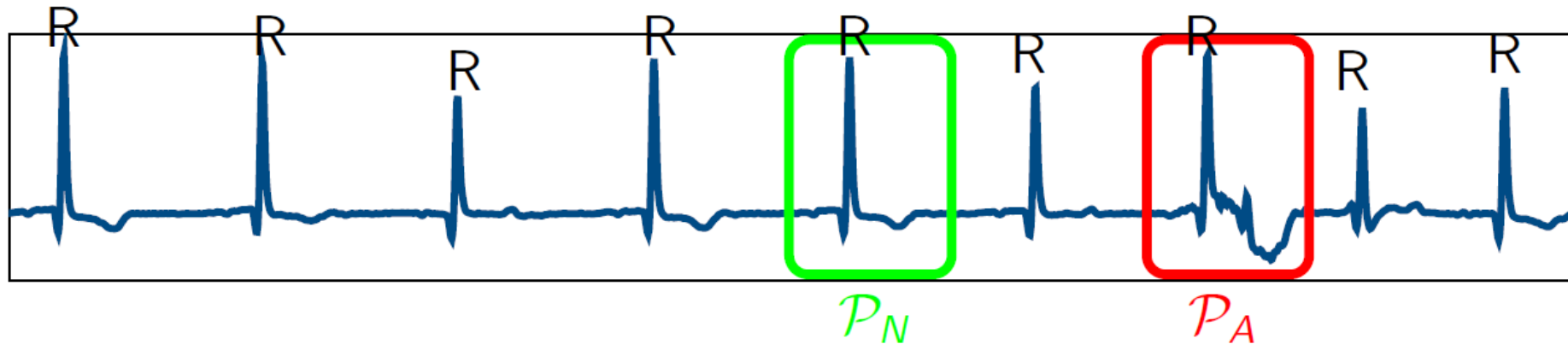
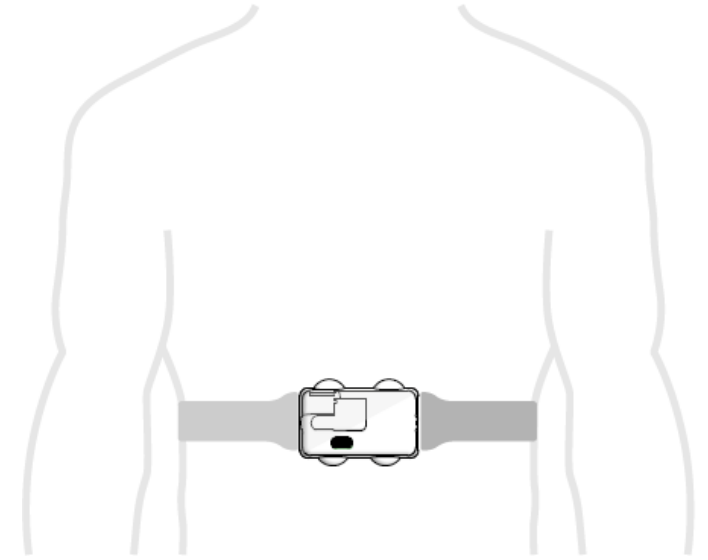




# Automatic and long term Ecg monitoring

Health monitoring / wearable devices:

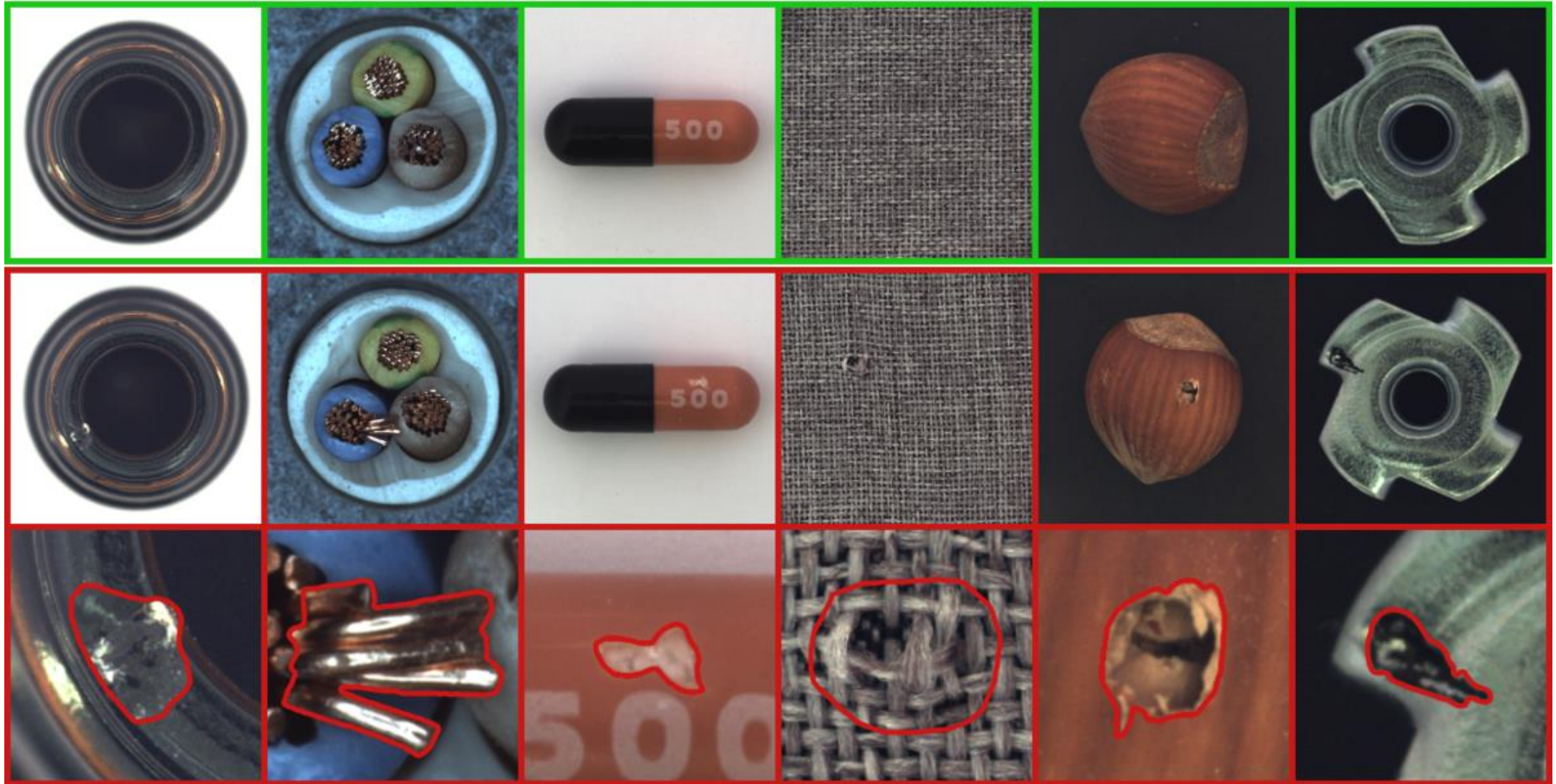
Automatically analyze ECG tracings to detect arrhythmias or incorrect device positioning



# Anomalous activities detection in videos

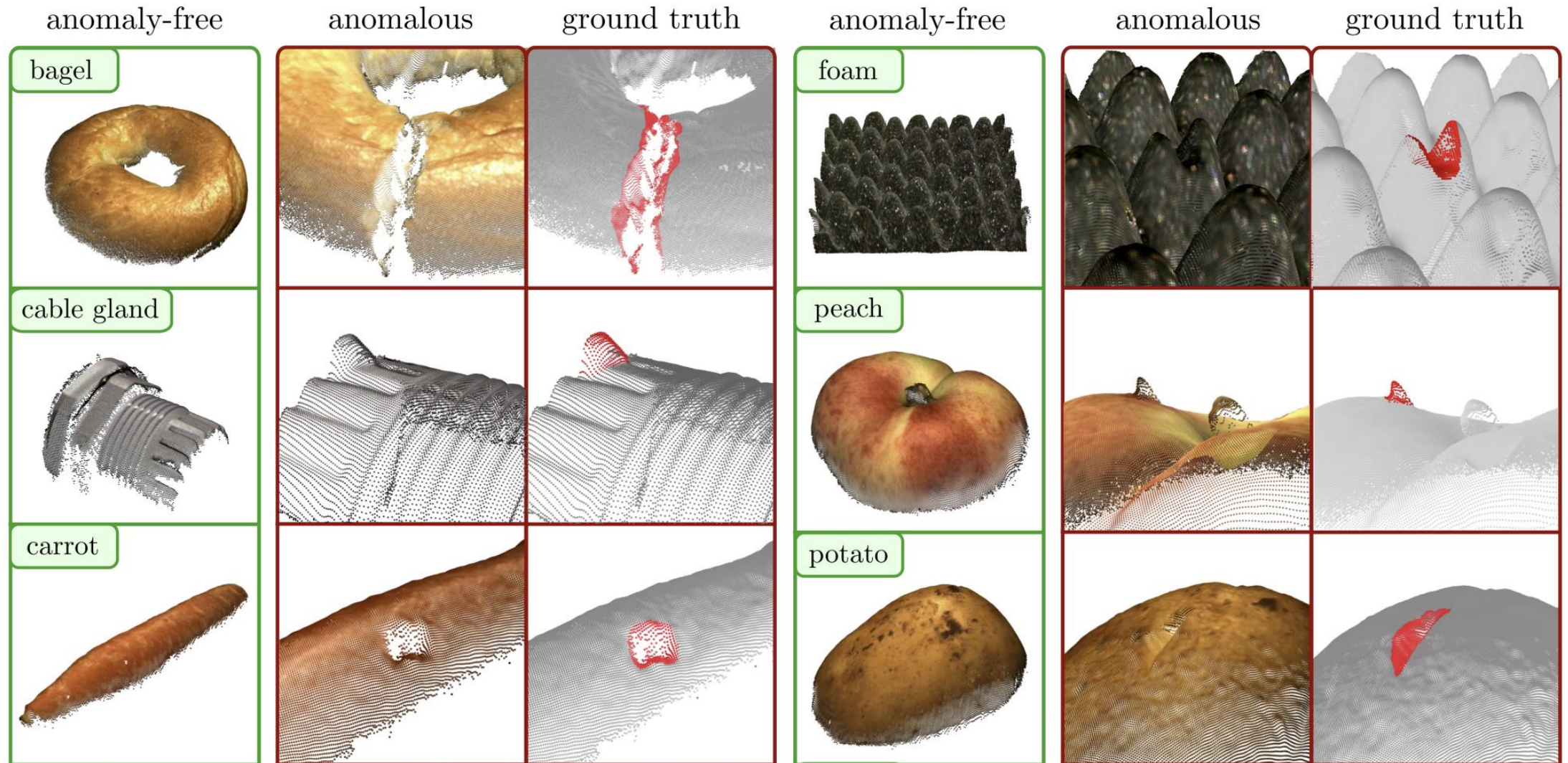


# Anomaly Detection for automatic quality control



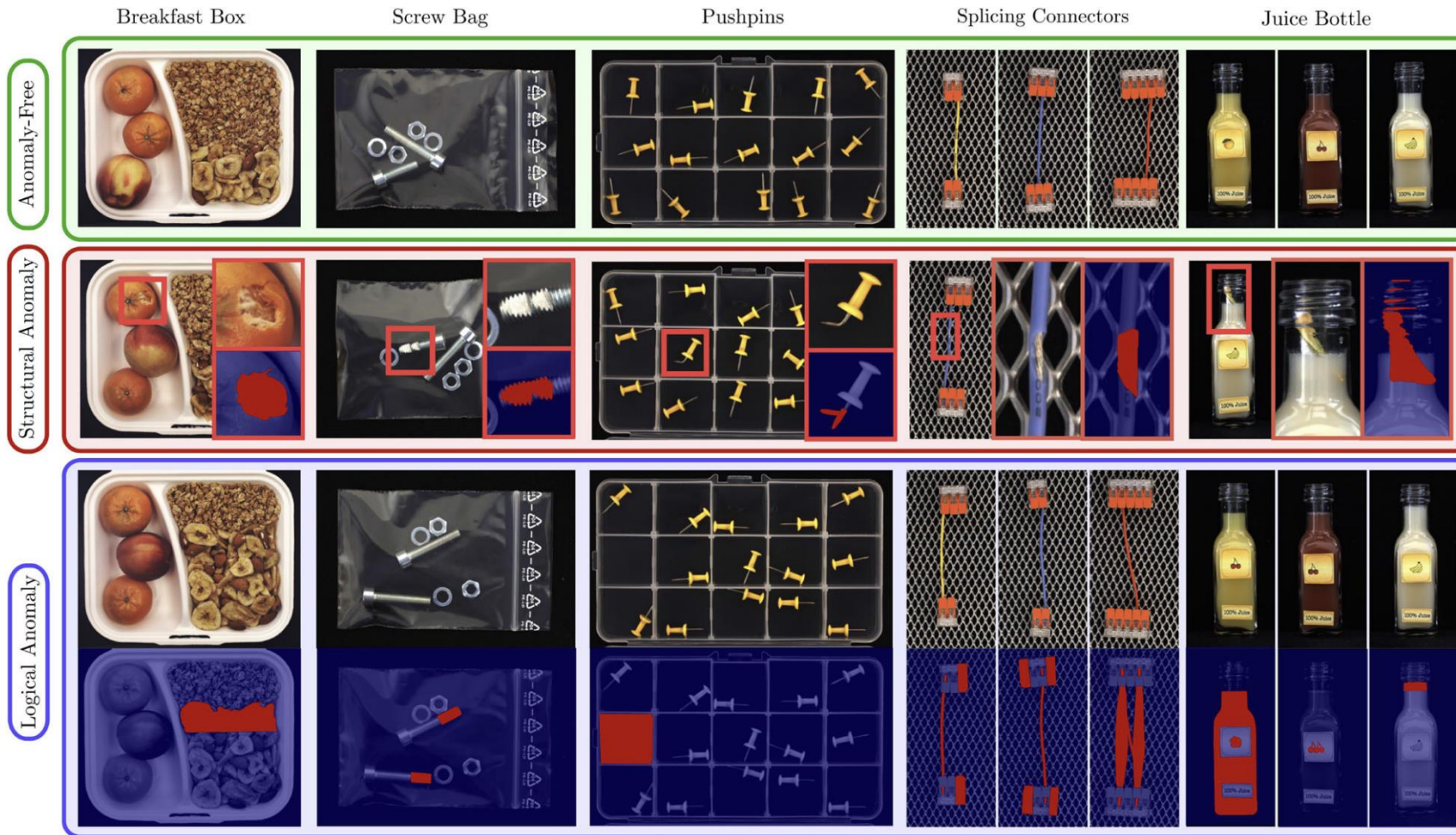


# 3D Anomaly Detection





# Logical constraints anomaly detection



**Fig. 3** Example images of the MVTec LOCO AD dataset for each of the five dataset categories. Each category contains anomaly-free train, validation, and test images. Additional test images contain various structural and logical anomalies. Pixel-precise ground truth annotations are provided for all anomalies

# The Problem Formulation

Anomaly Detection in Images

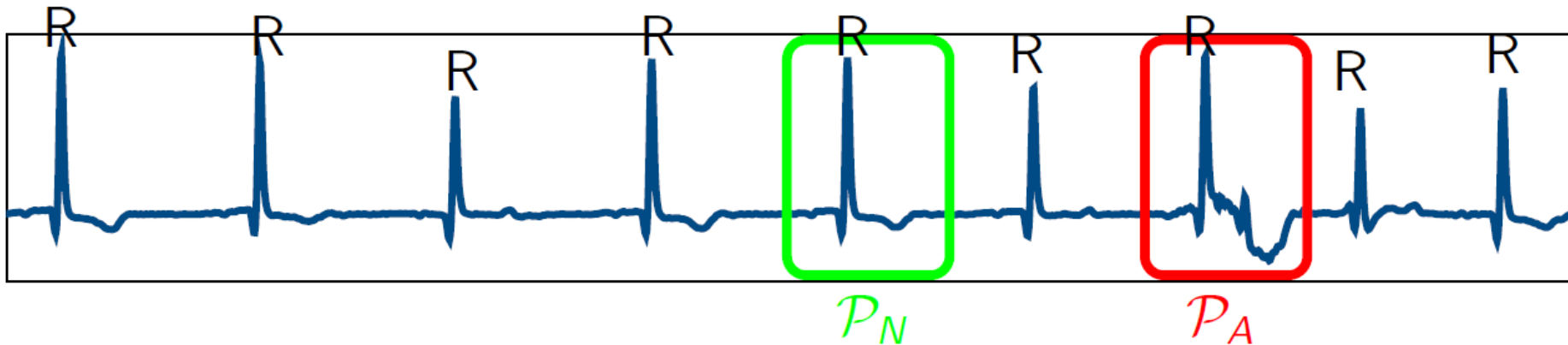


# Anomalies

*“Anomalies are patterns in data that do not conform to a well defined notion of normal behavior”*

Thus:

- Normal data are generated from a **stationary process**  $\mathcal{P}_N$
- **Anomalies** are from a **different process**  $\mathcal{P}_A \neq \mathcal{P}_N$

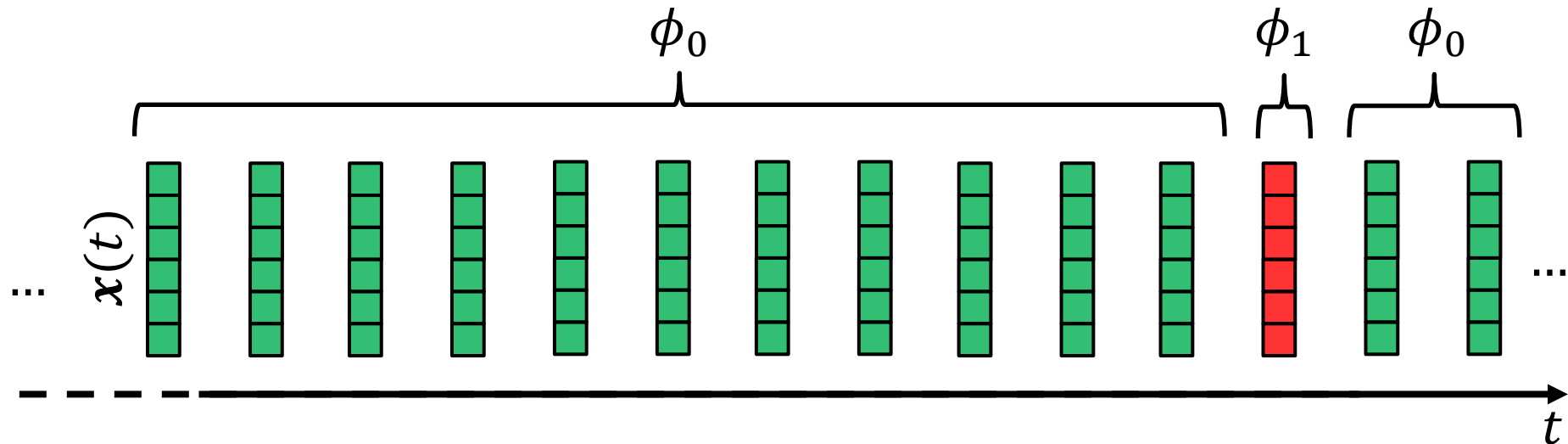


# Anomalies

*“Anomalies are patterns in data that do not conform to a well defined notion of normal behavior”*

Thus:

- **Normal data** are vectors drawn from a **stationary distribution**  $\phi_0$
- **Anomalies** are vectors drawn from a **different distribution**  $\phi_1 \neq \phi_0$



# Anomalies

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Examples of Anomalies:

- **Frauds** in the stream of all the credit card transactions
- **Arrhythmias** in ECG tracings
- **Defective regions in an image**, which do not conform a reference pattern
- Anomalies might appear as **spurious** elements, and are typically the **most informative** samples in the stream



# Problem formulation: Anomaly Detection in images

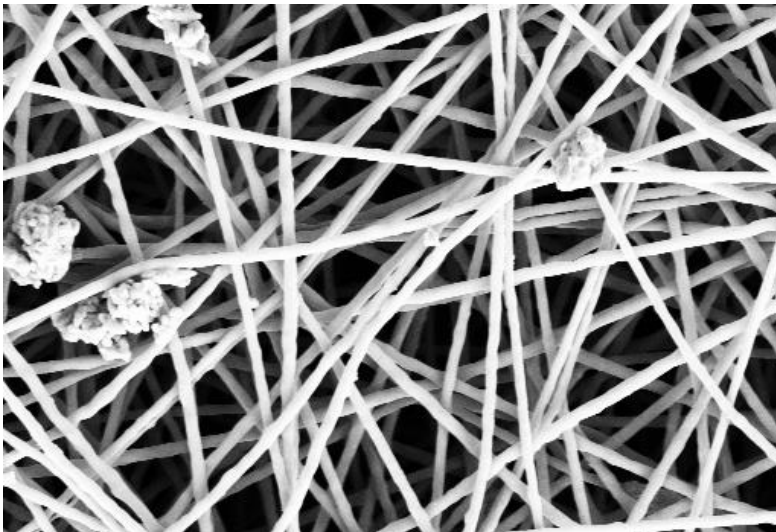
Let  $s$  be an image defined over the pixel domain  $\mathcal{X} \subset \mathbb{Z}^2$ ,

let  $c \in \mathcal{X}$  be a pixel and  $s(c)$  the corresponding intensity.

Our goal is to **locate any anomalous region** in  $s$ , i.e. **estimating the unknown anomaly mask  $\Omega$**  defined as

$$\Omega(c) = \begin{cases} 0 & \text{if } c \text{ falls in a normal region} \\ 1 & \text{if } c \text{ falls in an anomalous region} \end{cases}$$

$s$

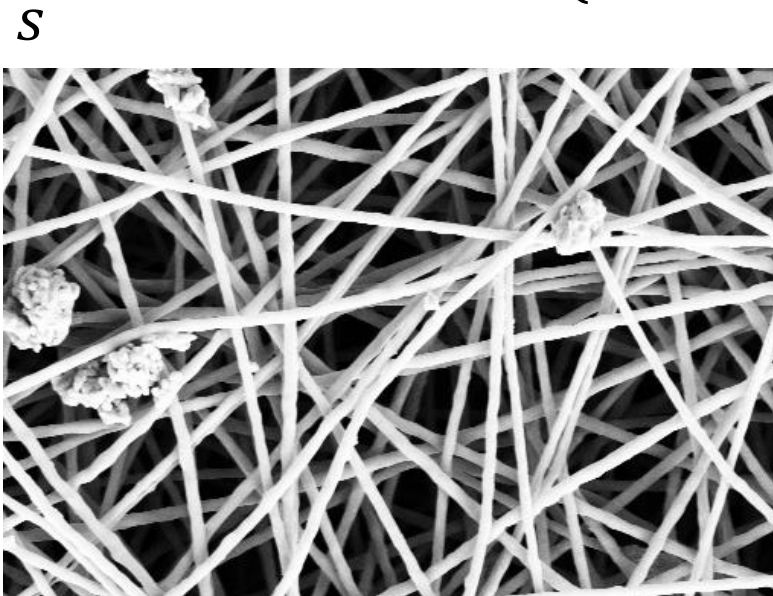


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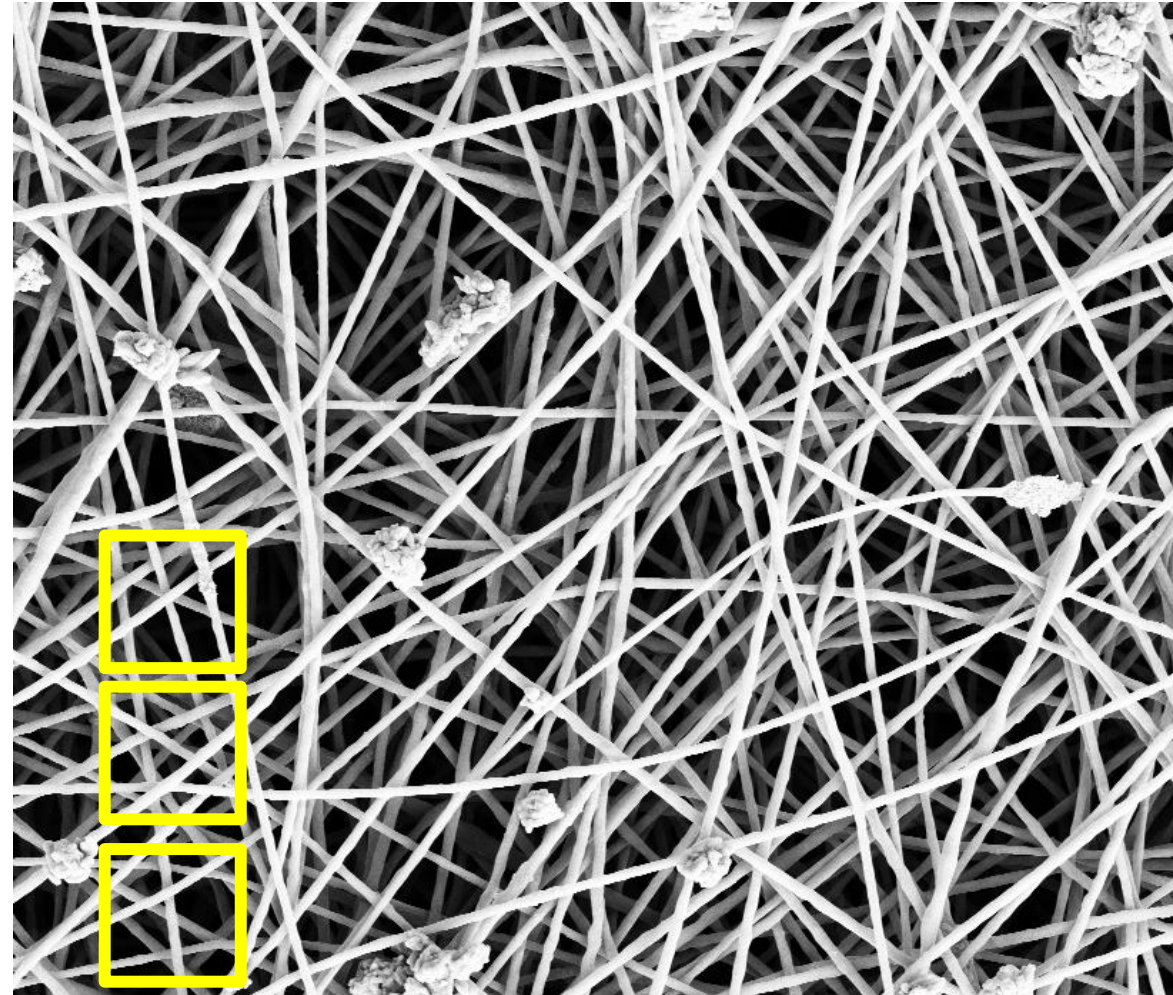


# Patch-wise Anomaly detection

The goal not determining whether the whole image is normal or anomalous, but **locate/segment possible anomalies**

Therefore, it is convenient to

1. **Analyze the image patch-wise**
2. Isolate regions containing patches that are detected as anomalies



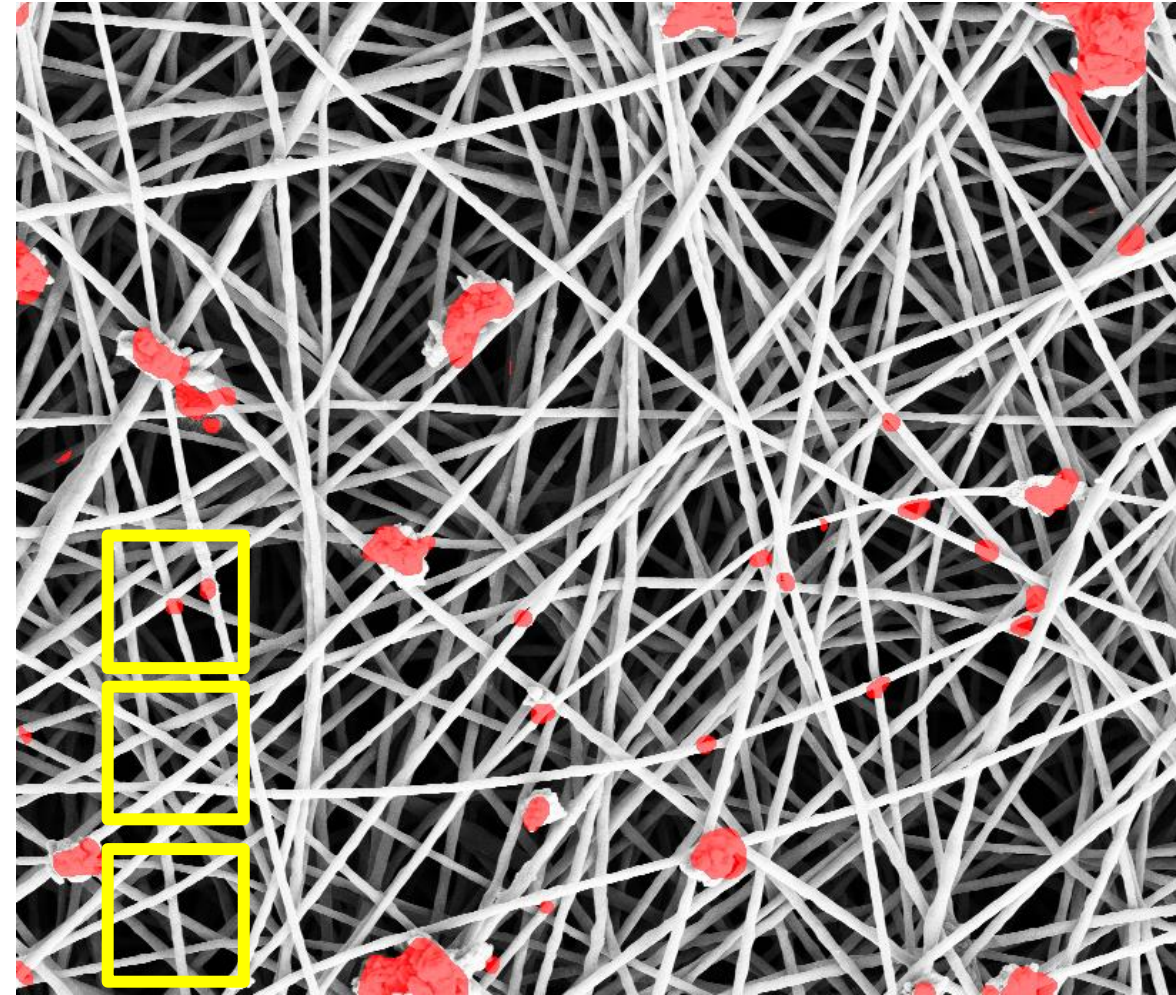


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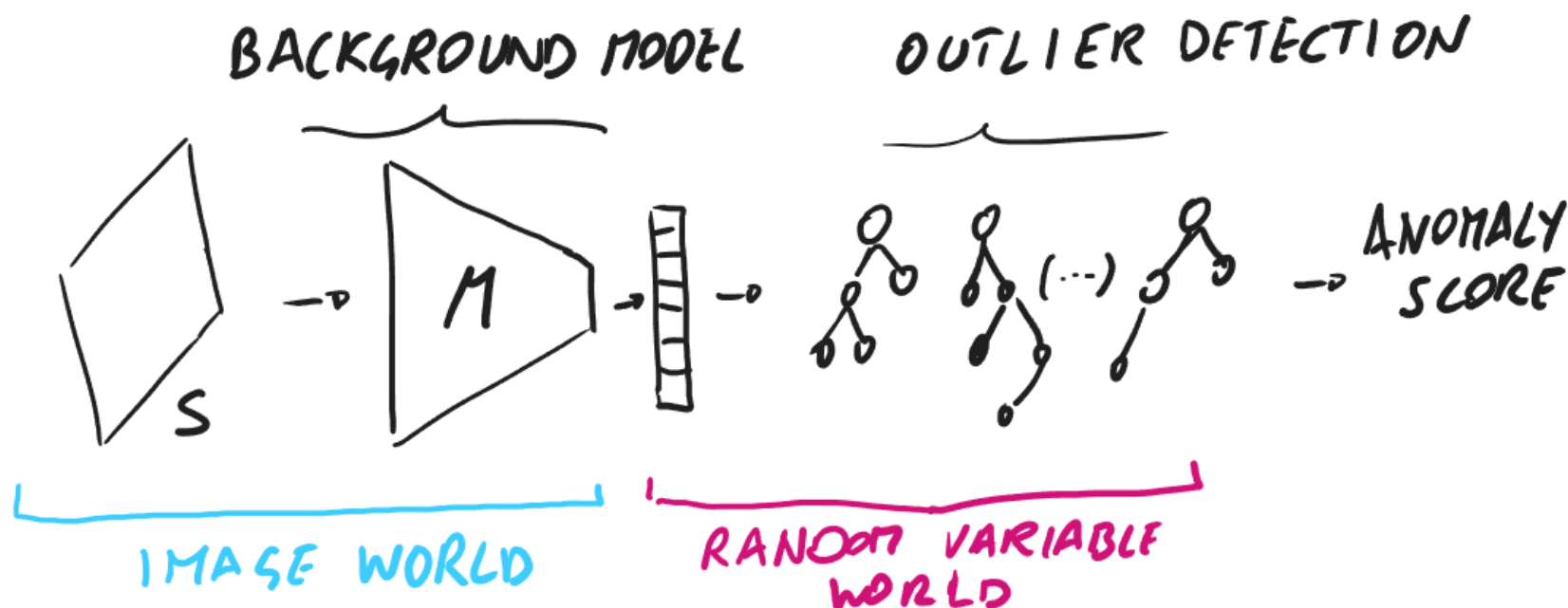
# The Mainstream Solutions

Anomaly Detection in Images

# The three major ingredients

Most detection algorithms have three major ingredients:

- The **background model**  $\mathcal{M}$ , learned from normal data
- The **statistic / anomaly score**:  $\text{err}(\mathbf{s})$ ,  $\mathcal{L}(\mathbf{s})$ ,  $\mathcal{A}(\mathbf{s})$ , ...
- **Decision rule** to detect, e.g.  $\text{err}(\mathbf{s}) \geq \gamma$  possibly controlling the FPR, as in other statistical detection methods

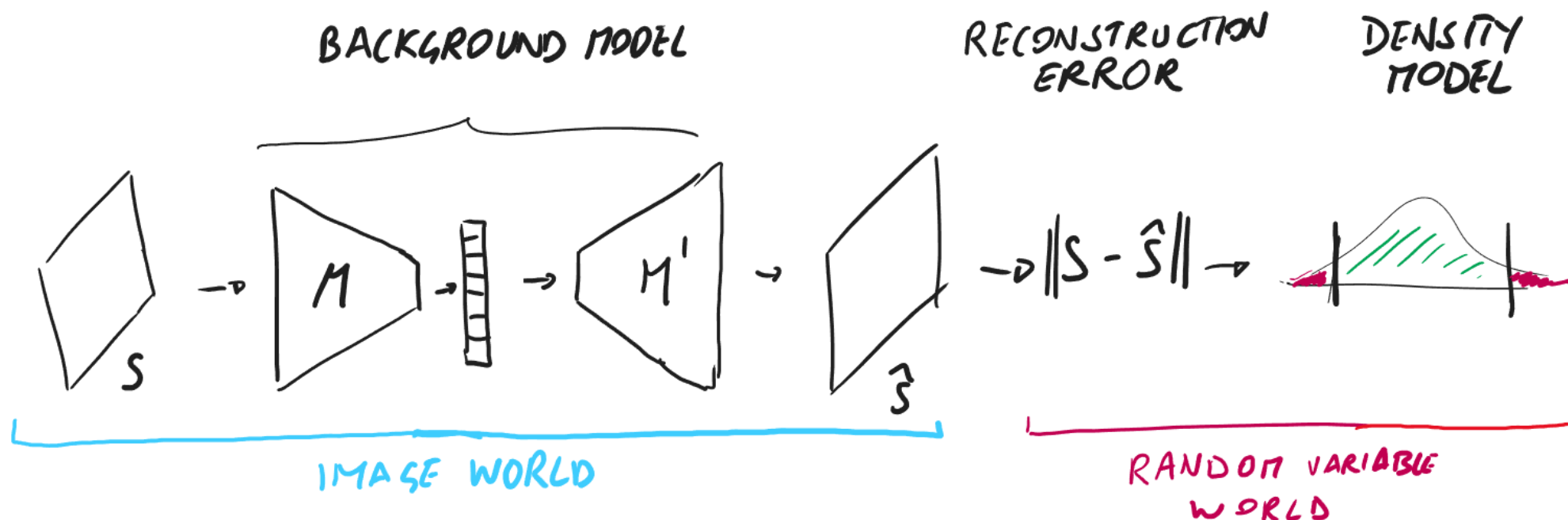




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Diego Carrera

# Anomaly Detection By Sparse Representations

Diego Carrera, Beatrice Rossi, Pasqualina Fragneto, Giacomo Boracchi *"Online Anomaly Detection for Long-Term ECG Monitoring using Wearable Devices"*, Pattern Recognition, 2019

Marco Longoni, Diego Carrera, Beatrice Rossi, Pasqualina Fragneto, Marco Pessione, Giacomo Boracchi, *"A Wearable Device for Online and Long-Term ECG Monitoring"* IJCAI 2018 - Demo Track

Carrera D., Rossi B., Fragneto P., and Boracchi G. *"Domain Adaptation for Online ECG Monitoring"* ICDM 2017,

Diego Carrera, Fabio Manganini, Giacomo Boracchi, Ettore Lanzarone, *"Defect Detection in SEM Images of Nanofibrous Materials"* IEEE TII, 2017

Diego Carrera, Giacomo Boracchi, Alessndro Foi and Brendt Wohlberg *"Scale-invariant Anomaly Detection With Multiscale Group-sparse Models"* IEEE ICIP 2016

# Dictionaries Yielding Sparse Representations

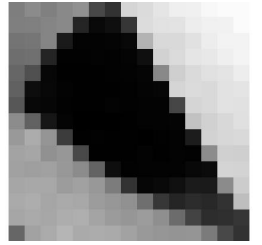
Dictionaries are just matrices!  $D \in \mathbb{R}^{n \times m}$

Each column is called *an atom*:

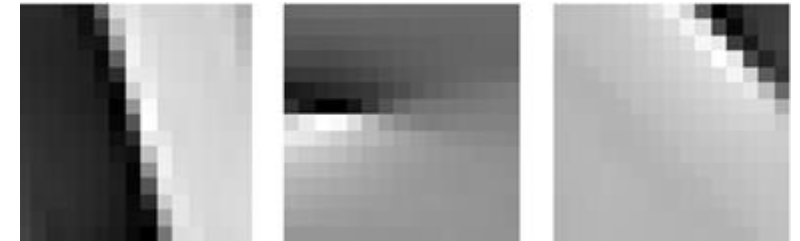
- lives in the input space
- it is one of the learned building blocks to reconstruct the input signal

**A good background model**

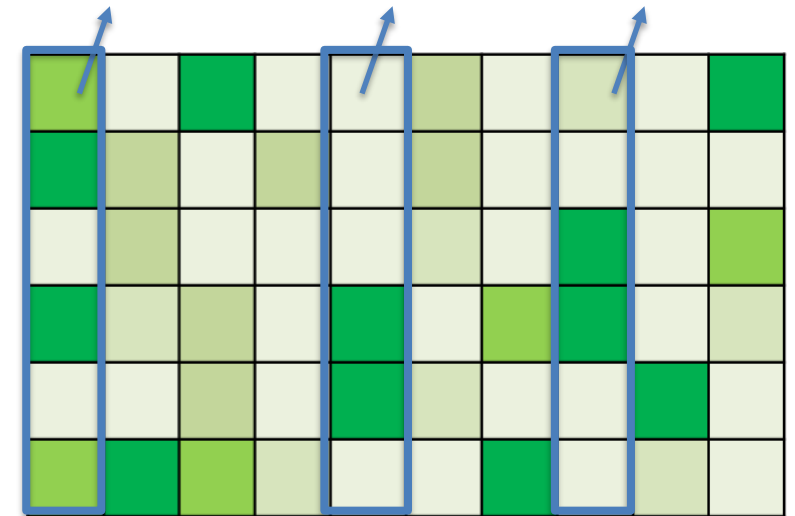
- Unsupervised models
- Easy to plug in a **change/anomaly detection** framework
- Easy to **adapt**
- Simple and interpretable



$S$



$D =$



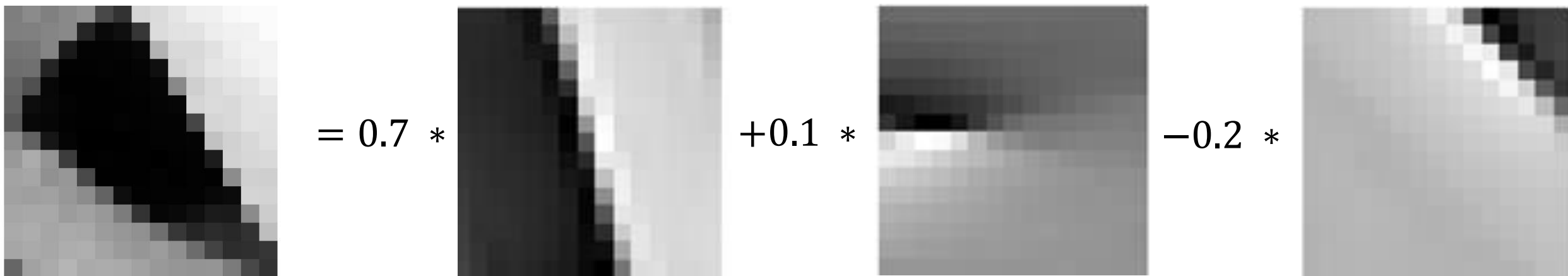


# Sparse Representations

Let  $\mathbf{s} \in \mathbb{R}^n$  be the input signal, a **sparse representation** is

$$\mathbf{s} = \sum_{i=1}^M x_i \mathbf{d}_i,$$

A sparse representation is a **linear combination of few dictionary atoms**  $\{\mathbf{d}_i\}$

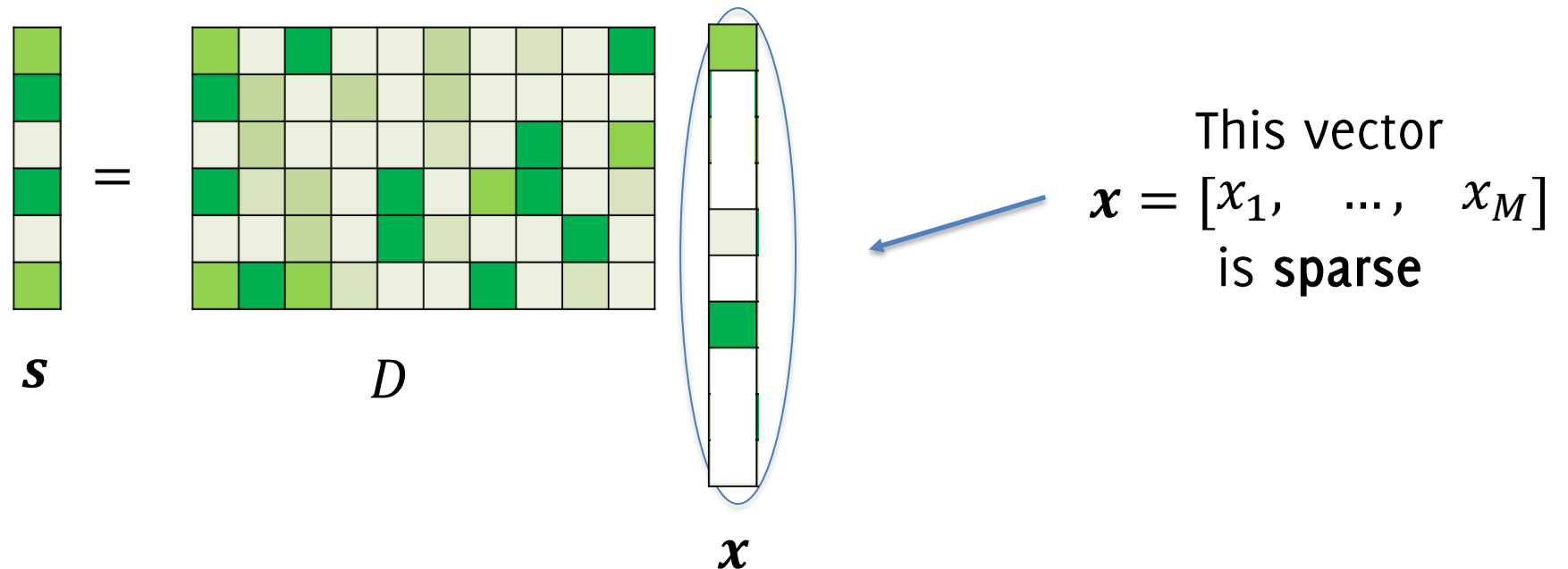


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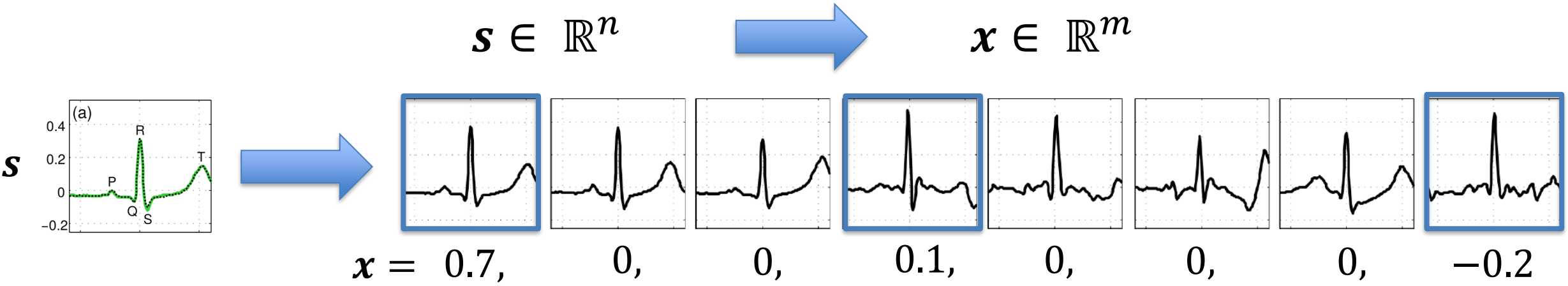
$$\mathbf{s} = \sum_{i=1}^M x_i \mathbf{d}_i = D\mathbf{x}$$

A sparse representation is a **linear combination of few dictionary atoms  $\{\mathbf{d}_i\}$**  and  $\|\mathbf{x}\|_0 < L$ , i.e. only a few coefficients are nonzero, i.e.  $\mathbf{x}$  is **sparse**.

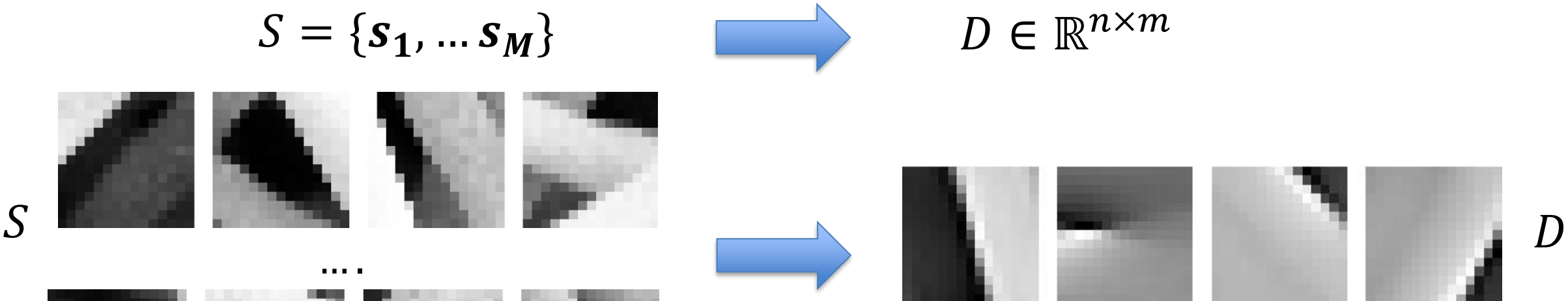


# Sparse Coding and Dictionary Learning

**Sprase Coding:** computing the sparse representation for an input signal  $s$  w.r.t.  $D$



**Dictionary Learning:** estimate  $D$  from a training set of signals  $S$

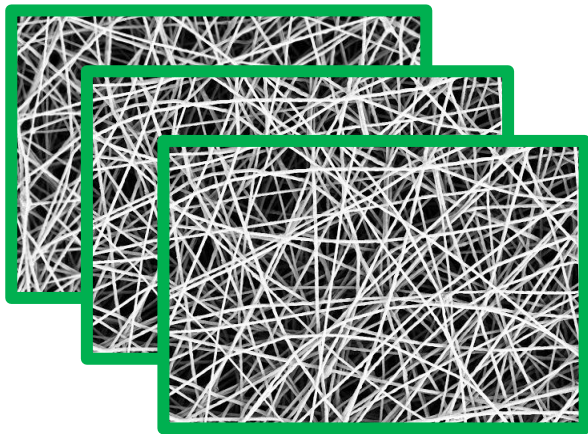




# Online Monitoring through Sparse Representations

Training:

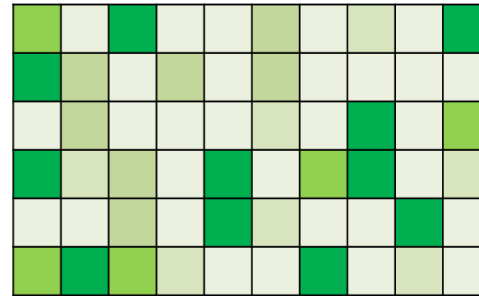
- Learn a dictionary  $D$  from a training set  $S$  containing **normal** instances



$TR$



Dictionary  
Learning

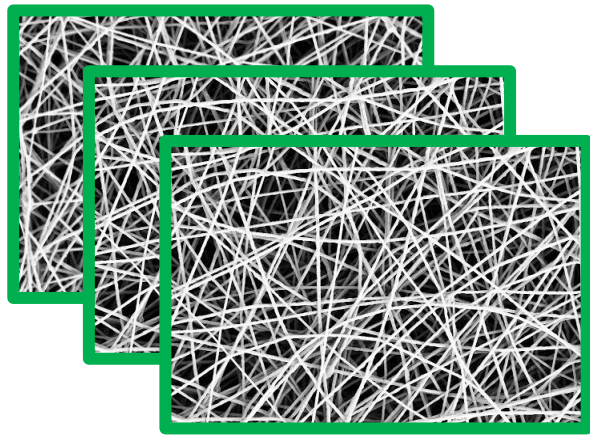


$D$

# Online Monitoring through Sparse Representations

## Training:

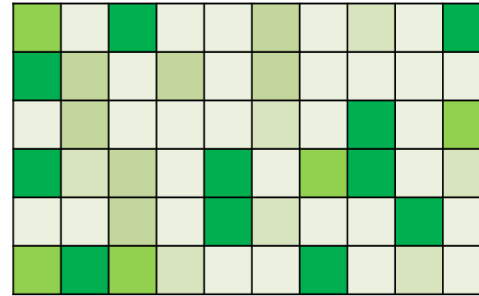
- Learn a dictionary  $D$  from a training set  $S$  containing normal instances
- Learn how normal data are reconstructed by  $D$



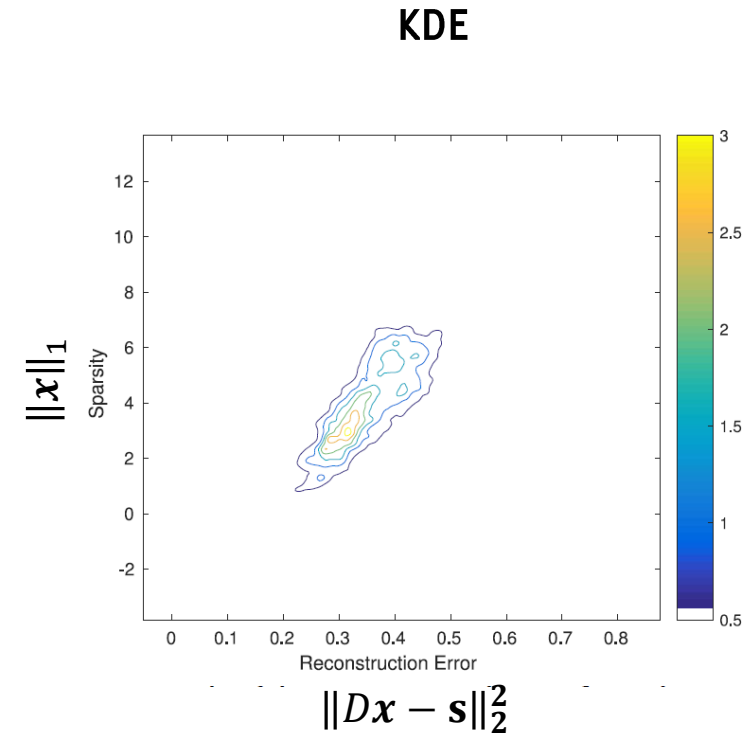
$TR$



Dictionary  
Learning



$D$



# Online Monitoring through Sparse Representations

## Training:

- Learn a dictionary  $D$  from a training set  $S$  containing **normal instances**
- Learn how normal data are reconstructed by  $D$

Anomalous



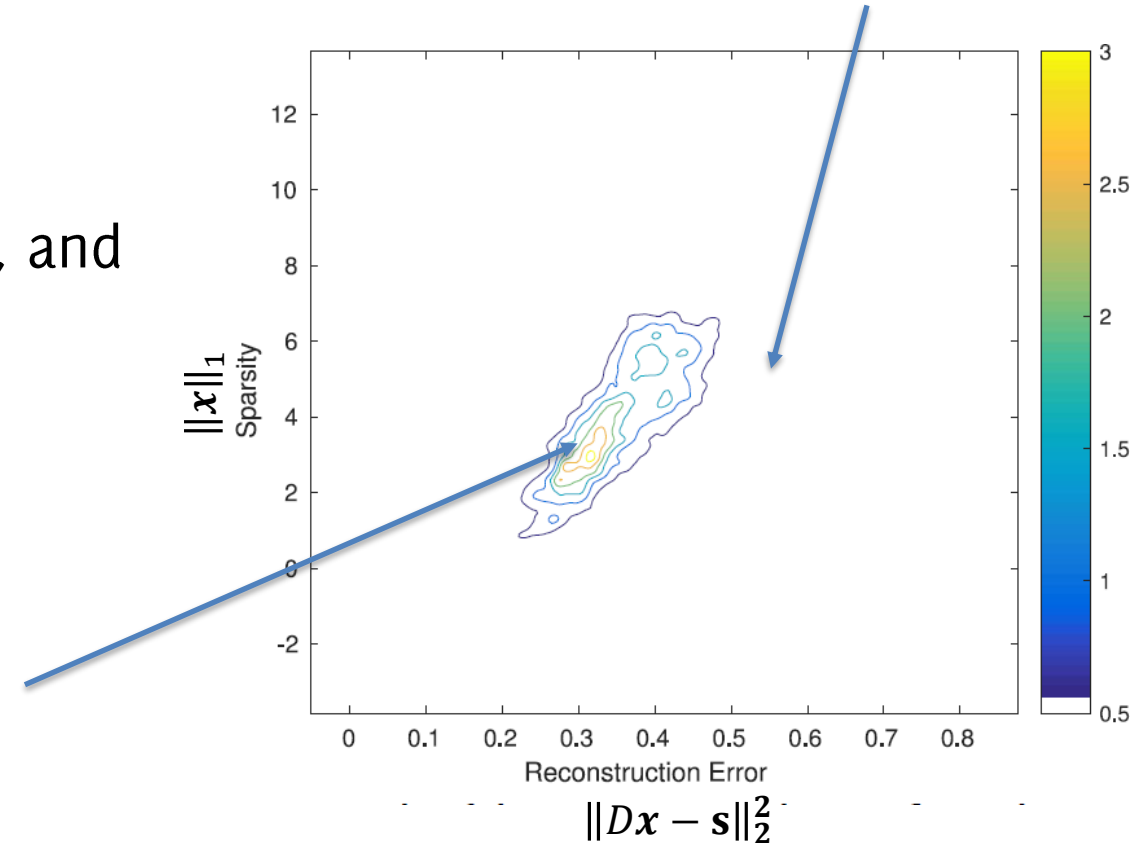
## Anomaly Detection:

**Sparse Coding:** encode each test signal  $\mathbf{s}$  w.r.t.  $D$ , and assess its conformance with  $D$ .

Check whether the representation is:

- Sparse  $\|\mathbf{x}\|_1$
- Accurate  $\|D\mathbf{x} - \mathbf{s}\|_2^2$

Normal

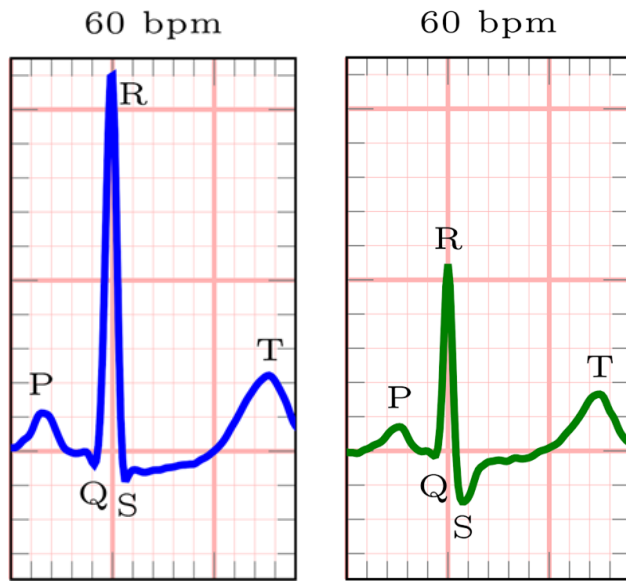




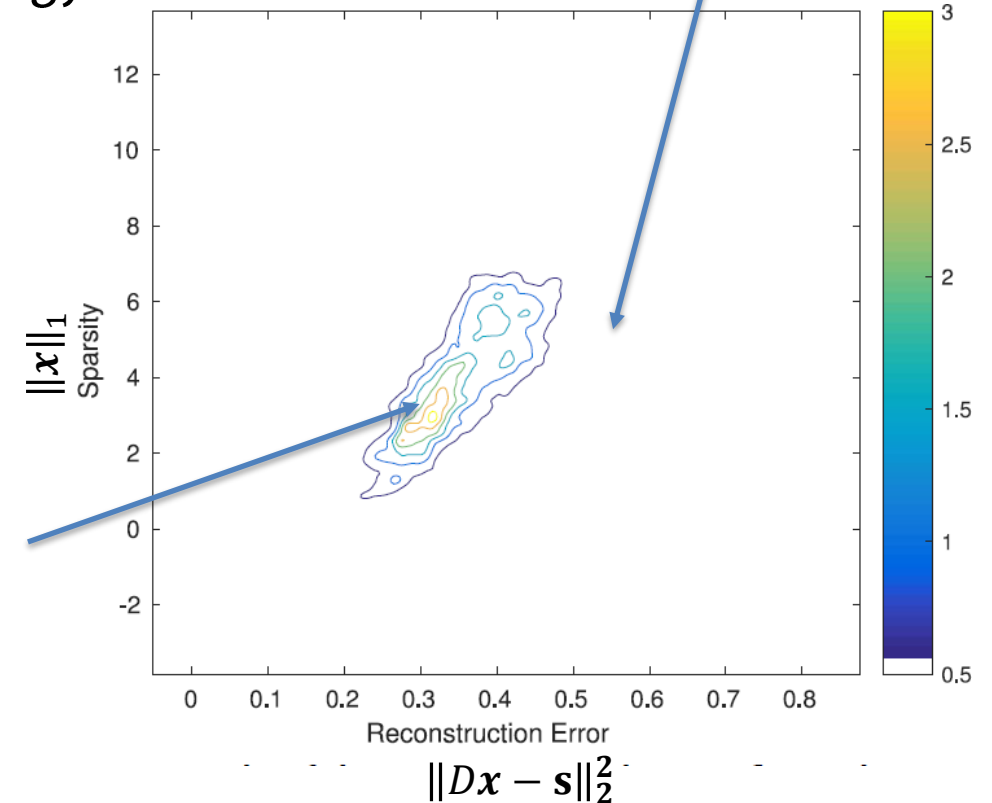
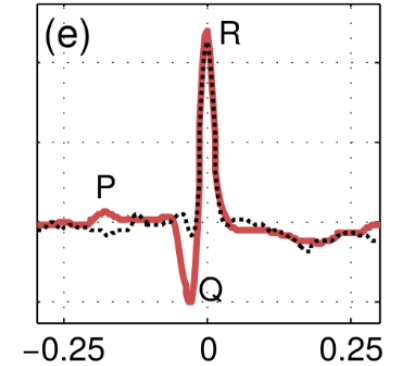
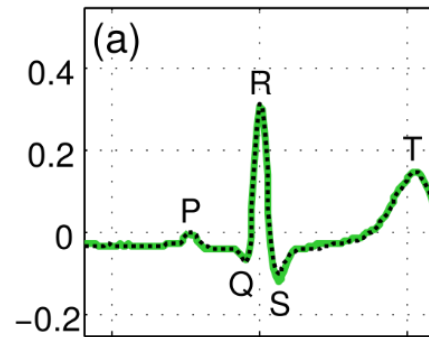
# Anomaly Detection in Heartbeats

The solution is **general** and can be customized to different scenarios, including ECG monitoring.

Here we learn user-specific dictionaries to detect arrhythmias as heartbeats that do not conform to the user morphology



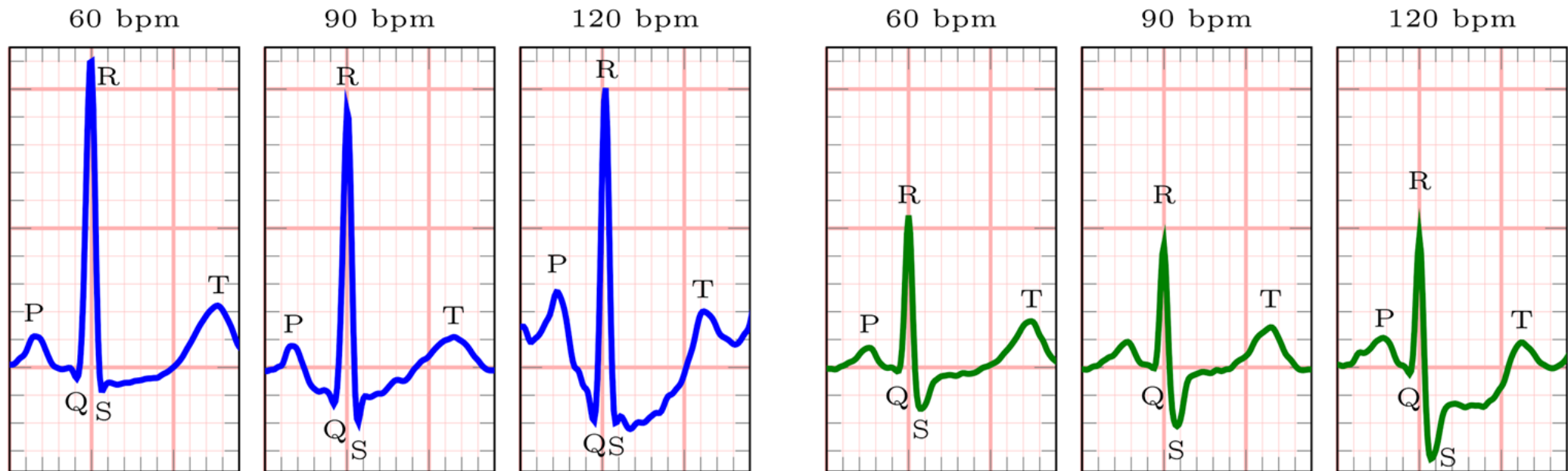
Different users feature different heartbeat morphology



# Domain Adaptation for Online ECG Monitoring

The issue:

- Dictionary has to be learned from each user
- ECG tracings for training can be only acquired in resting conditions
- During daily activities heart-rate changes and do not match the learned dictionary



The heartbeats get transformed when the heart rate changes:  
learned models have to be adapted according to the heart rate.

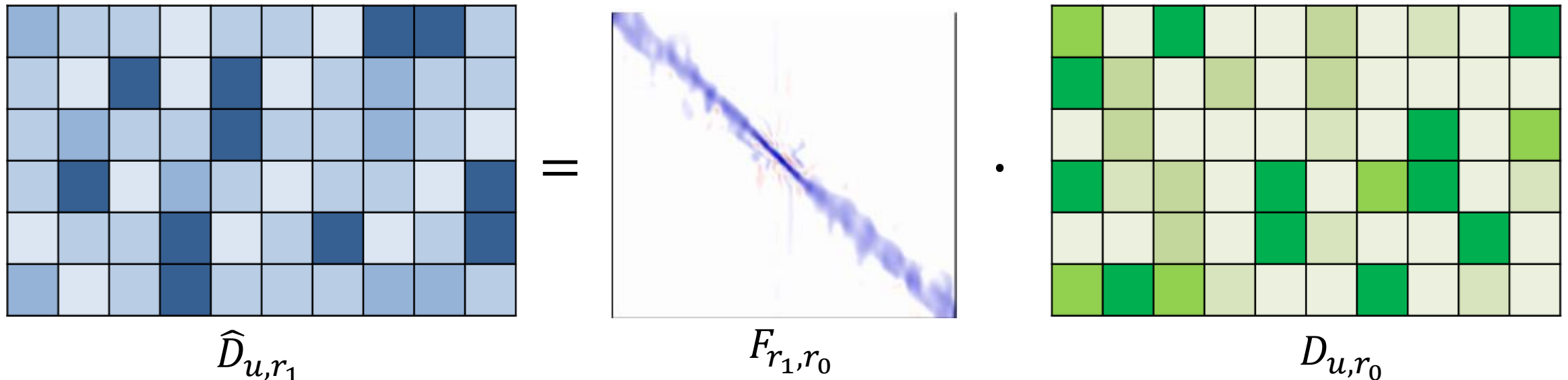
# Domain Adaptation for Online ECG Monitoring

We propose to design linear transformations  $F_{r_1, r_0}$  to adapt user-specific dictionaries

$$D_{u, r_1} = F_{r_1, r_0} \cdot D_{u, r_0}, \quad F_{r_0, r_1} \in \mathbb{R}^{m \times m}$$

Surprisingly these **transformations** can be **learned from a publicly available dataset** containing ECG recordings at different heart rates from several users

**User-independent transformations** enable accurate mapping of user-specific dictionaries





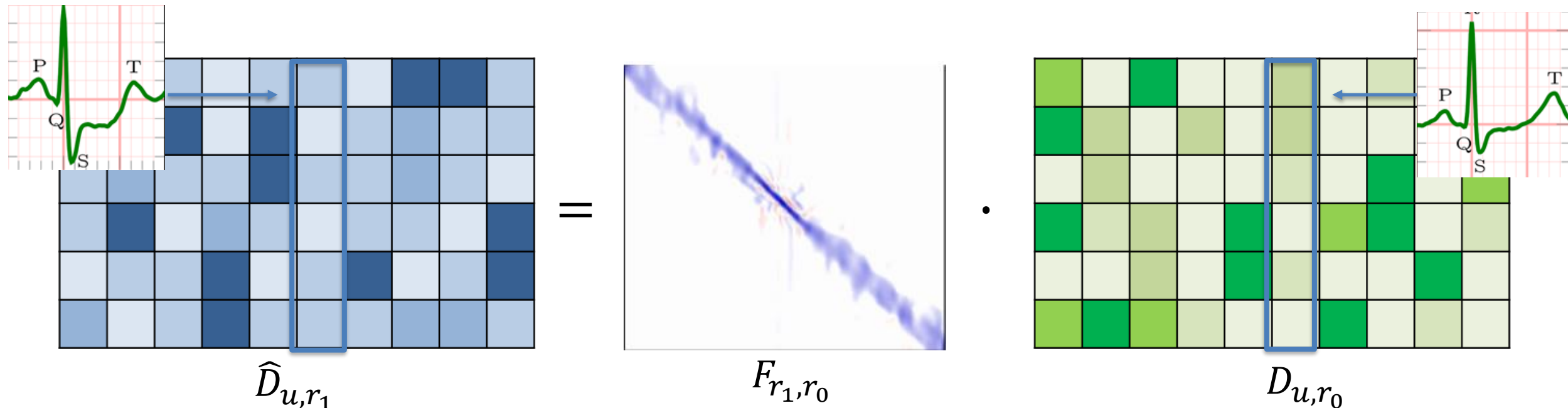
# Domain Adaptation for Online ECG Monitoring

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**User-independent transformations** enable accurate mapping of user-specific dictionaries



# ML: Long Term ECG monitoring in wearables

Efficient variant of the sparse coding has been implemented in an MCU, to enable **online and long term ECG monitoring**. Automatic **detection of anomalies** (e.g., arrhythmias) and **domain adaptation** to track heart rate variations. The model is based on a **learned and user-specific model** of heartbeats.



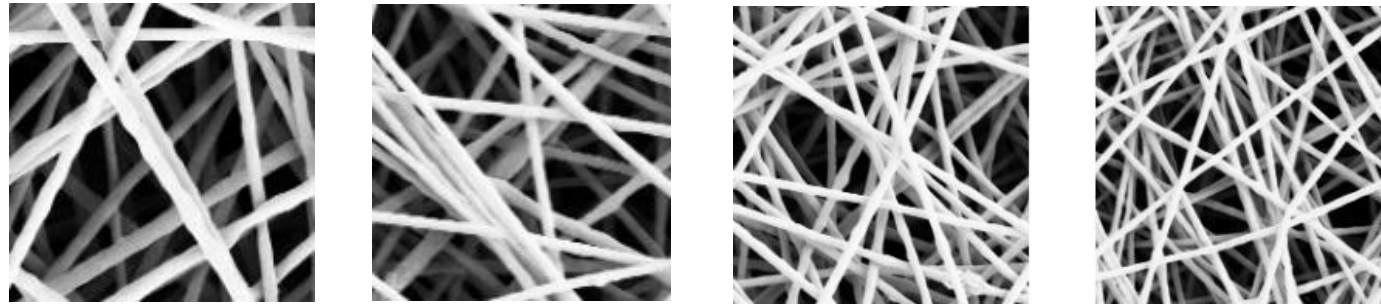
# Domain Adaptation on Quality Inspection

## The Issue:

- SEM images can be acquired at different zooming levels

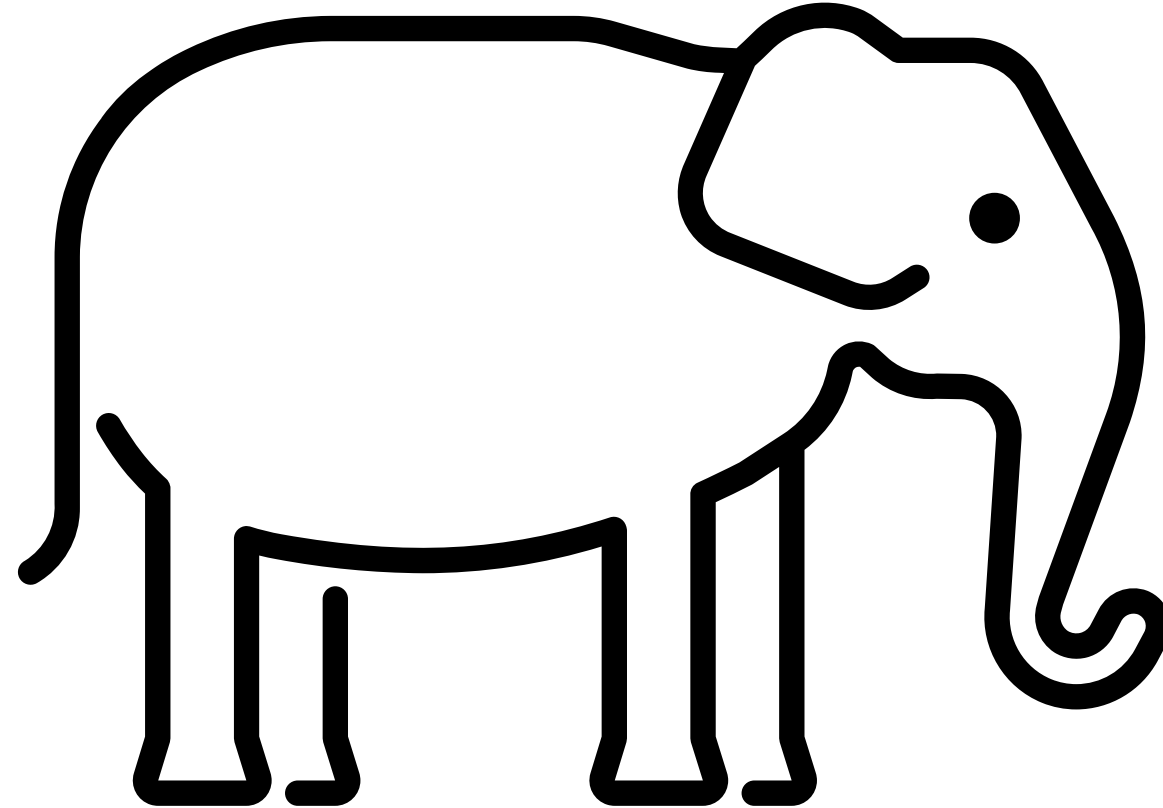
## Solution:

- **Synthetically generate** training images at **different zooming levels**
- Learn a dictionary for each scale
- Combine all the learned dictionaries in a **multiscale dictionary  $D$**
- Perform **sparse-coding** including a penalized, **group sparsity term**



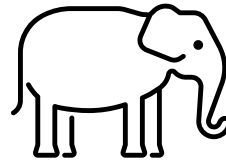
$$D = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \end{bmatrix}$$

The Elephant in the Room...

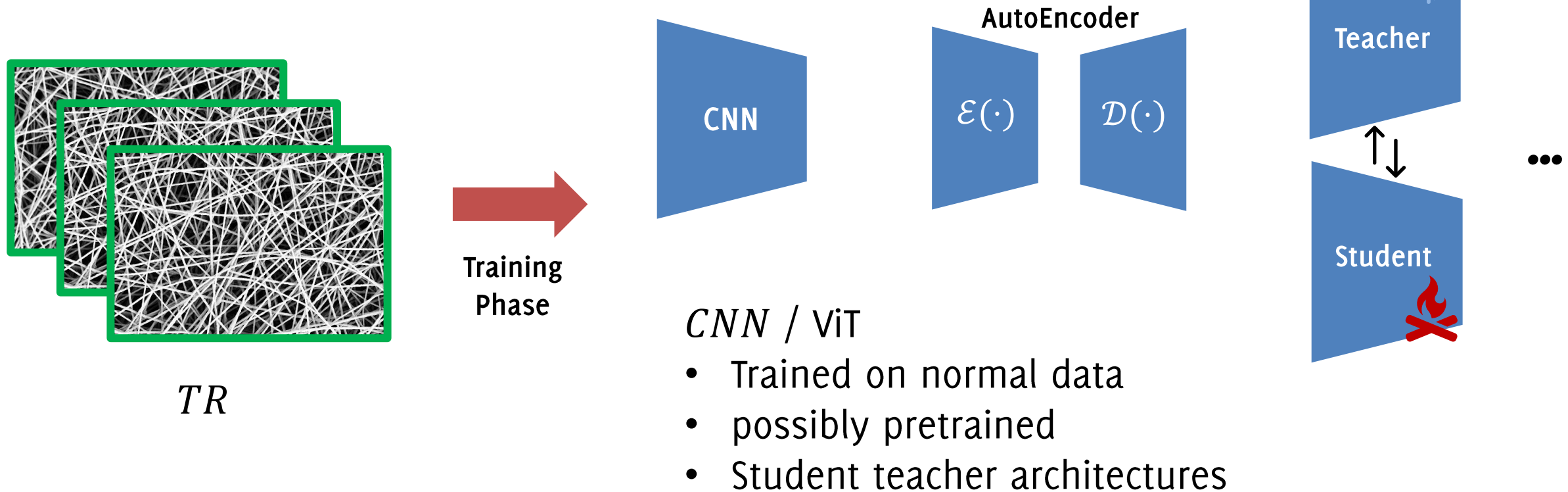




# Anomaly Detection by Deep Learning Models



In 2018 the «deep learning» tsunami meets Anomaly Detection



Napoletano P., Piccoli F., Schettini R., "Anomaly Detection in Nanofibrous Materials by CNN-Based Self-Similarity", Sensors 2018

Zong et al, "Deep Autoencoding Gaussian Mixture Model for Unsupervised Anomaly Detection", ICLR 2018

Sabokrou, Mohammad, et al. "Deep-anomaly: Fully convolutional neural network for fast anomaly detection in crowded scenes." CVIU 2018

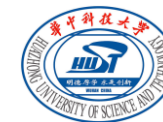
Burlina, Philippe, Neil Joshi, and I. Wang. "Where's Wally now? Deep generative and discriminative embeddings for novelty detection." CVPR 2019

Defard, Thomas, et al. "PADIM: a patch distribution modeling framework for anomaly detection and localization." ICPR 2020

Bergmann, Paul, et al. "Uninformed students: Student-teacher anomaly detection with discriminative latent embeddings." CVPR 2020



*Yunkang Cao*



# Zero-Shot Anomaly Detection

Yunkang Cao, Jiangning Zhang , Luca Frittoli, Yuqi Cheng, Weiming Shen, and Giacomo Boracchi  
*“AdaCLIP: Adapting CLIP with Hybrid Learnable Prompts for Zero-Shot Anomaly Detection” ECCV, 2024*

# Zero Shot Deep Learning

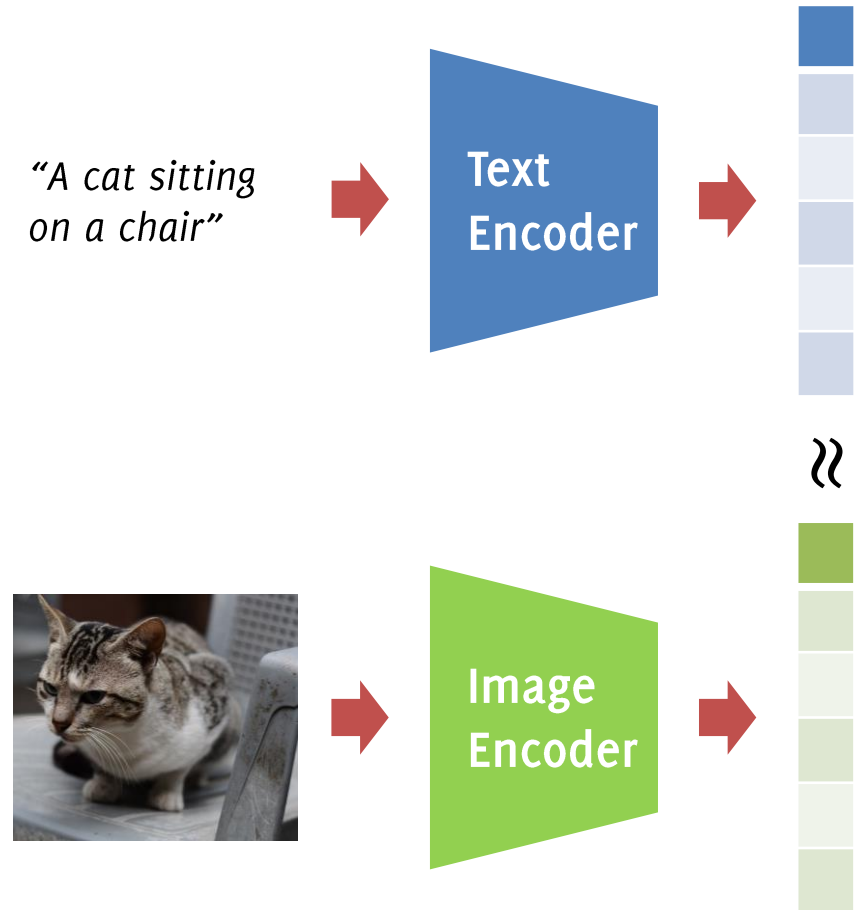
*Learning perception from the **supervision** contained in natural language paired with images.*

**CLIP:** a Text encoder and an Image encoder projecting latent representations in the same space.

The latent representations of a text and of an image are nearby when the two portraits similar content

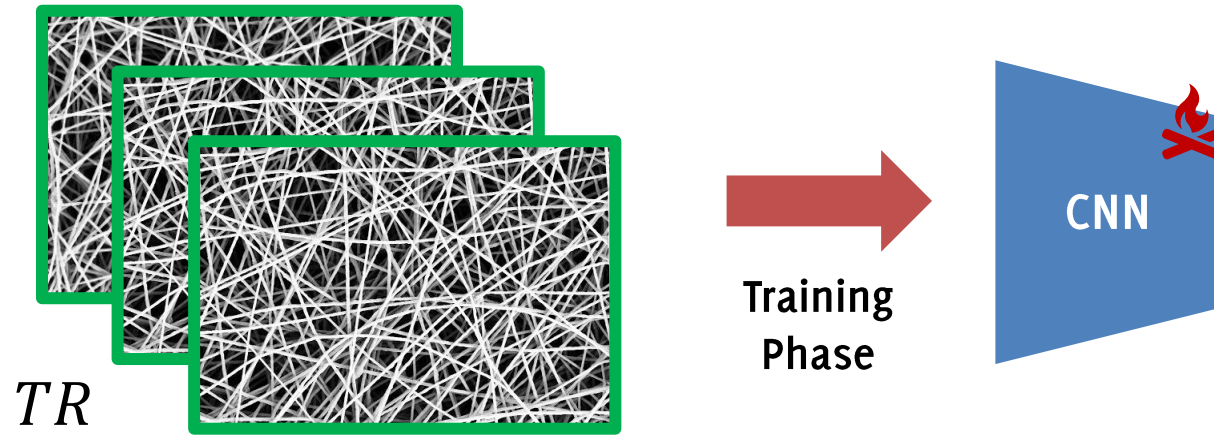
## Zero-Shot CLIP

- Take pre-trained CLIP
- Consider the class names as text  $\{t_i\}$
- Classify an image  $I$  by associating it to the text  $\{t_i\}$  that is closest to  $I$  according to CLIP.



# The Zero-Shot Paradigm in Anomaly Detection

Most AD methods require normal samples to train a (typically unsupervised) DL model



**“Cold Start” limitation: need to gather data before starting monitoring**

However, pre-trained **Vision-Language Models** “*know*” the visual pattern of defects.  
**Testing images may exhibit universal patterns**, either normal or anomalous, that VLMs can identify.

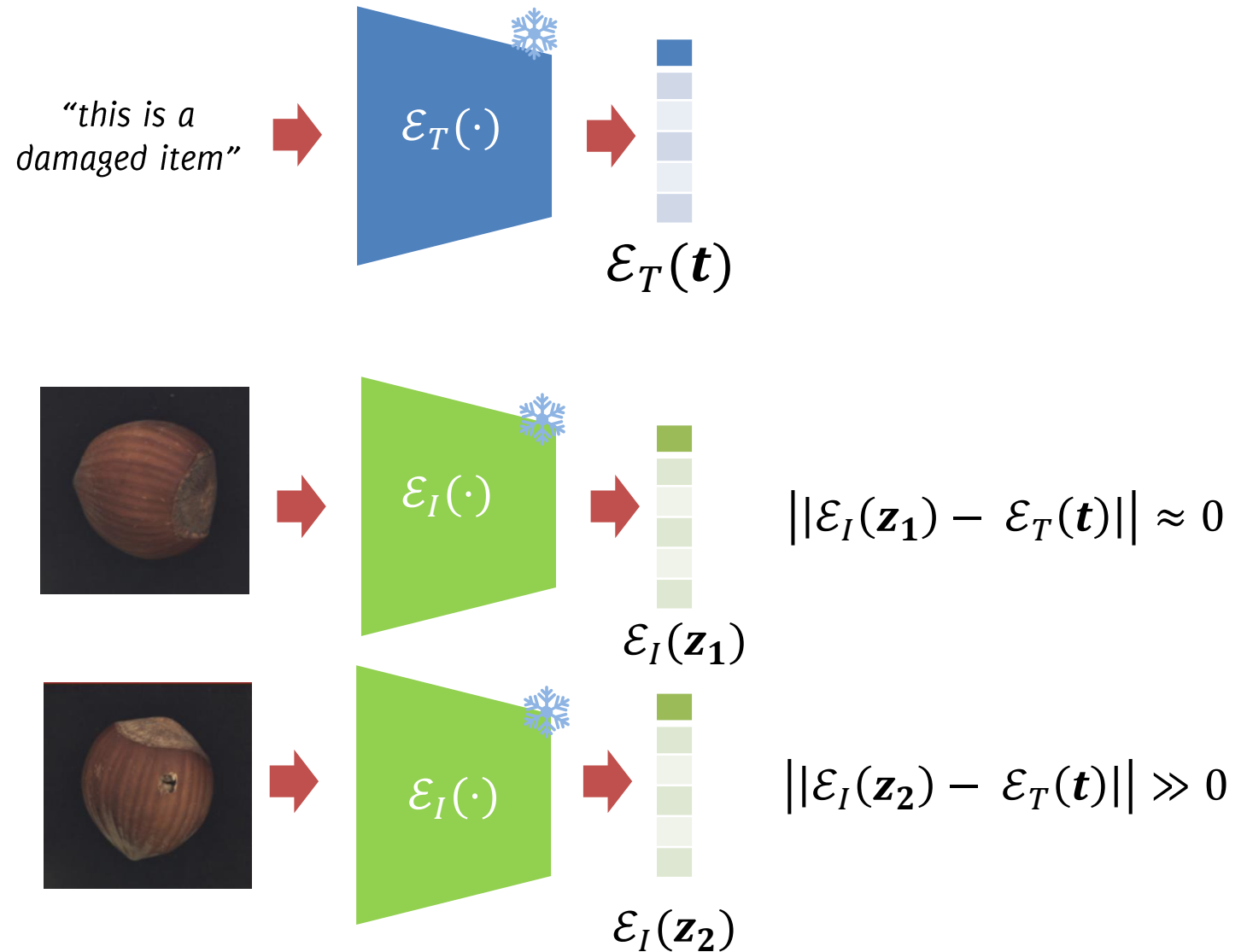


# The Zero-Shot Paradigm in Anomaly Detection

**Zero-shot AD:** Enable detecting anomalies even without normal images for training.

## Rationale behind Zero-shot AD:

- Adopt a pretrained CLIP Image and Text Encoders
- Query for “defective”, “broken” or “damaged” prompts
- Detect as anomalous images that are close to these prompts in the latent space.
- Possibly adopt “normal” text prompts as well



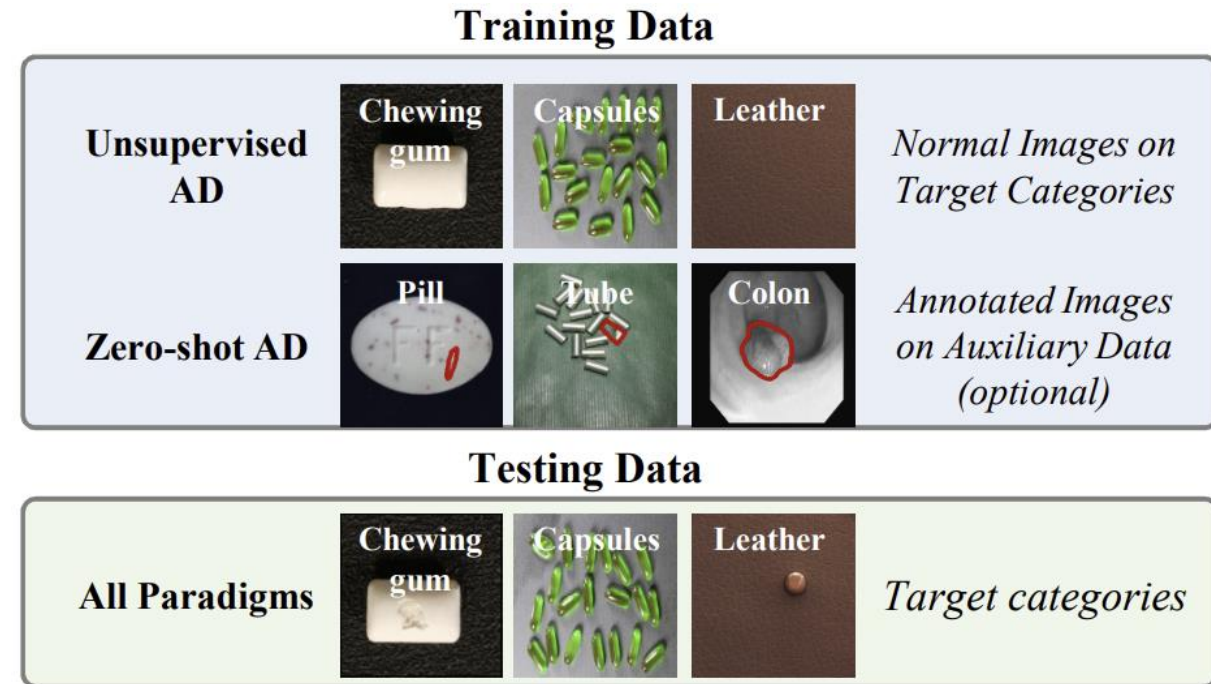
# AdaClip

## Remarks:

- CLIP is trained on **natural image-text**, not specialized for anomaly detection.
- There is a large **availability** of **defect detection annotated datasets**.

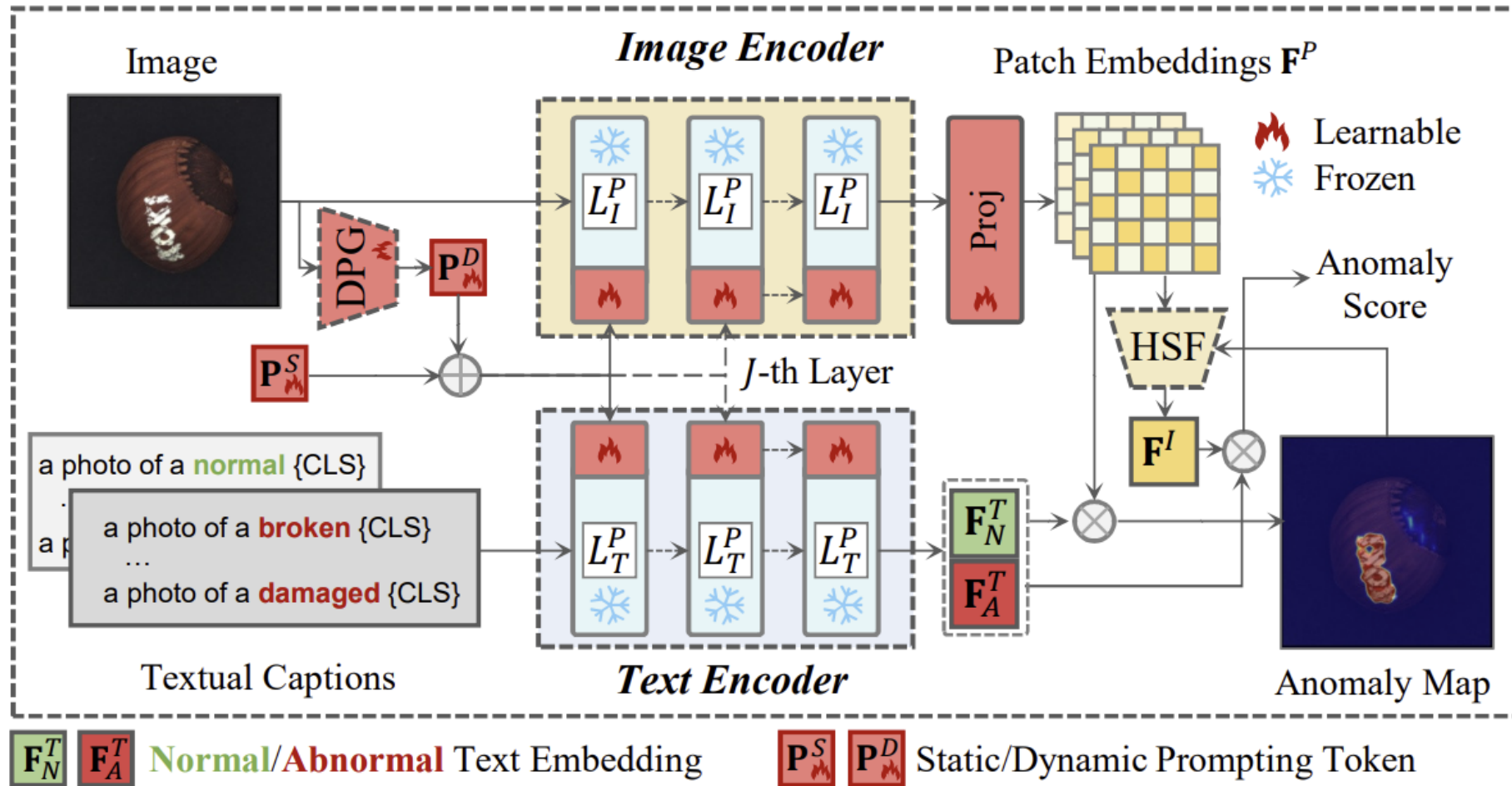
## Idea behind AdaCLIP:

- Leverage defect-detection datasets as auxiliary annotated data to **fine tune learnable prompts** into a zero-shot framework from CLIP.
- Adaptation adheres to the zero-shot learning paradigm, as long as **testing images do not belong to categories** presented in the **auxiliary AD dataset**.



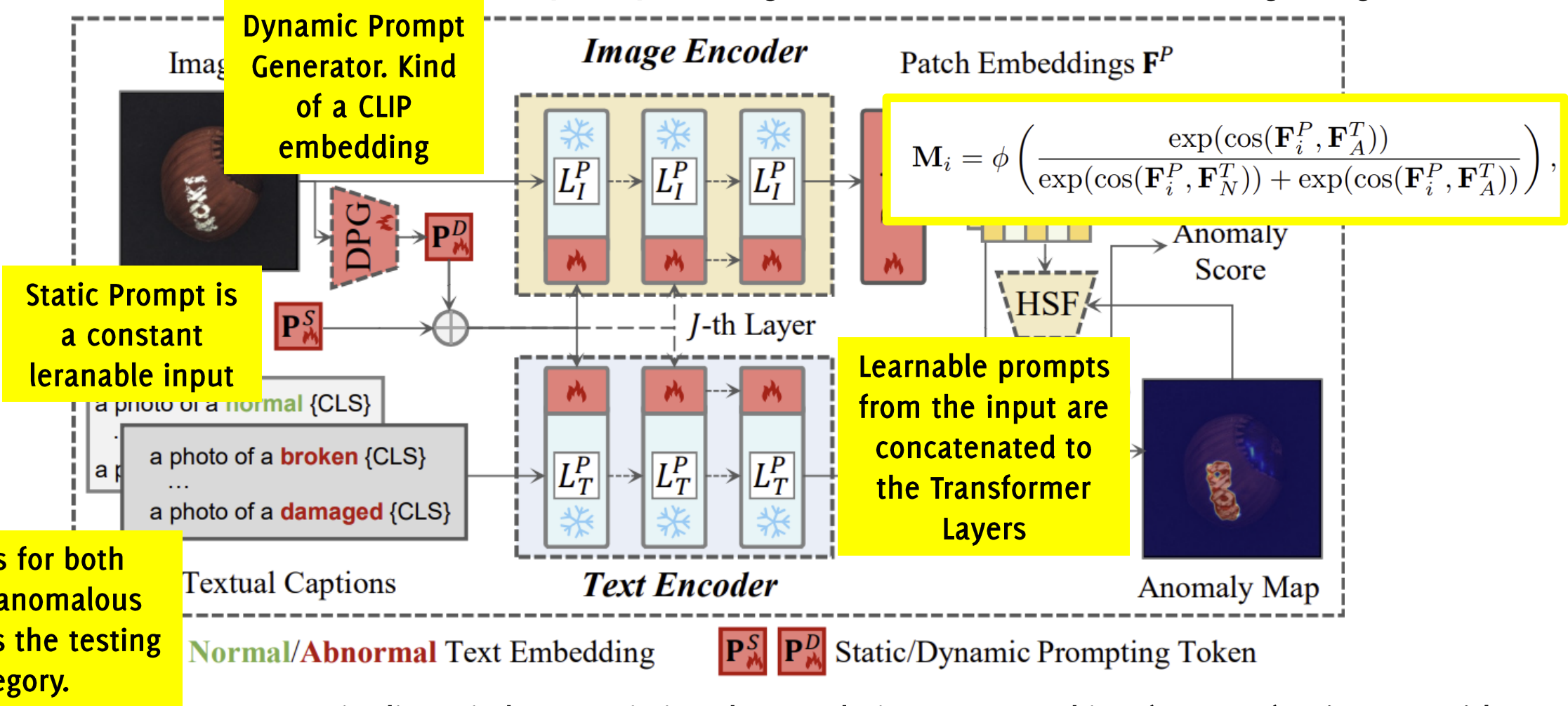
# AdaClip

AdaCLIP fine tunes on auxiliary datasets **projection and prompting layers**. **Static prompts** are shared across all images and **dynamic prompts** are generated based on the testing image



# AdaClip

AdaCLIP fine tunes on auxiliary datasets **projection and prompting layers**. **Static prompts** are shared across all images and **dynamic prompts** are generated based on the testing image







*Filippo Leveni*



*Luca Magri*

# Structure-based Anomaly Detection

Filippo Leveni, Luca Magri, Cesare Alippi, Giacomo Boracchi *“Preference Isolation Forest for Structure-based Anomaly Detection”* Under Review

Filippo Leveni, Luca Magri, Cesare Alippi and Giacomo Boracchi *“Hashing for Structure-based Anomaly Detection”*  
International Conference on Image Analysis and Processing (ICIAP) 2023 Best Paper Award

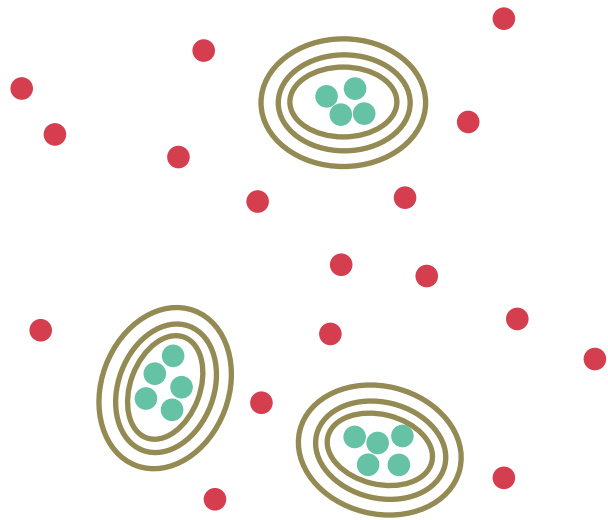
Filippo Leveni, Luca Magri, Giacomo Boracchi, Cesare Alippi, *“Anomaly detection via preference embedding”*  
International Conference on Pattern Recognition (ICPR) 2020,

# Structure-based Anomaly Detection

## Density-based Anomaly Detection

Anomalies are identified as samples that lie in *low-density regions*.

The **background model  $\mathcal{M}$**  is trained to extract i.i.d. indicators or anomaly scores, to move the AD problem in the random-variable settings.

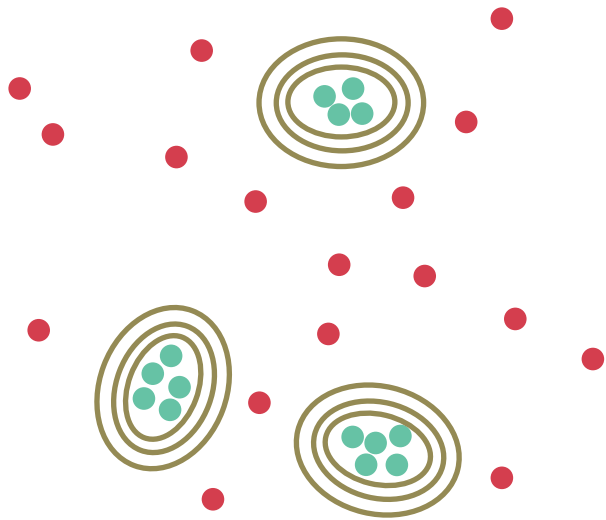


# Structure-based Anomaly Detection

## Density-based Anomaly Detection

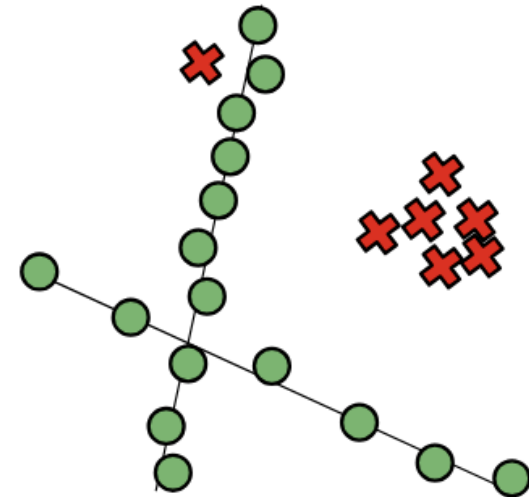
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## Structure-based Anomaly Detection

In some cases, anomalies are better identified as samples that do not conform to *low-dimensional manifolds*.

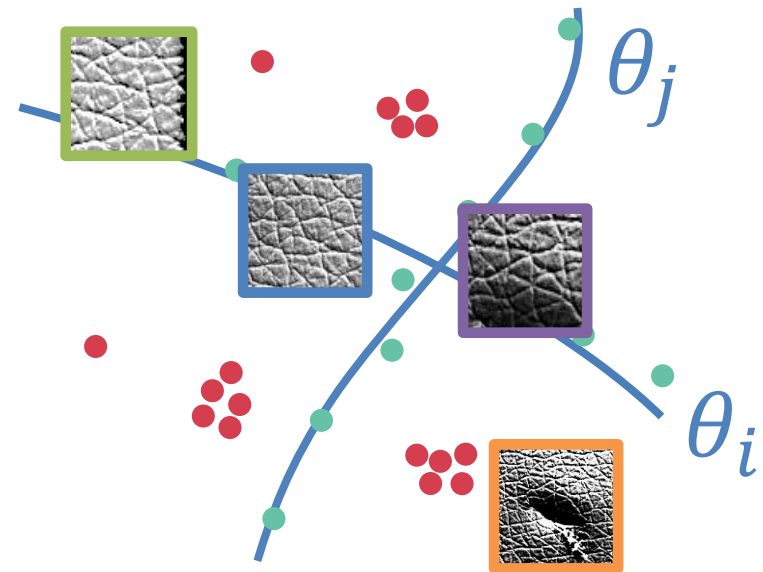
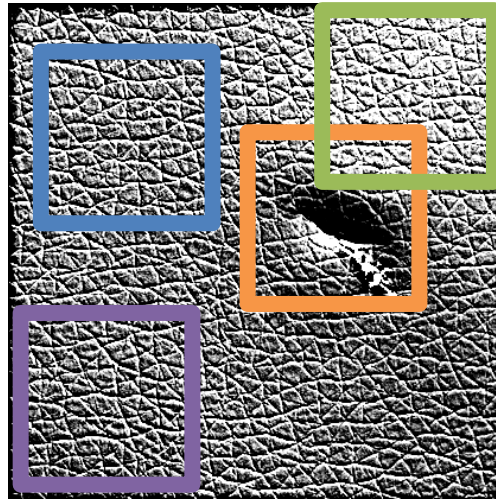
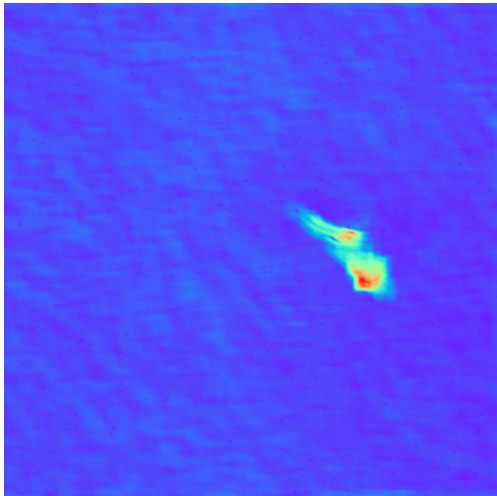


# Structure-based Anomaly Detection

## Structure-based Anomaly Detection

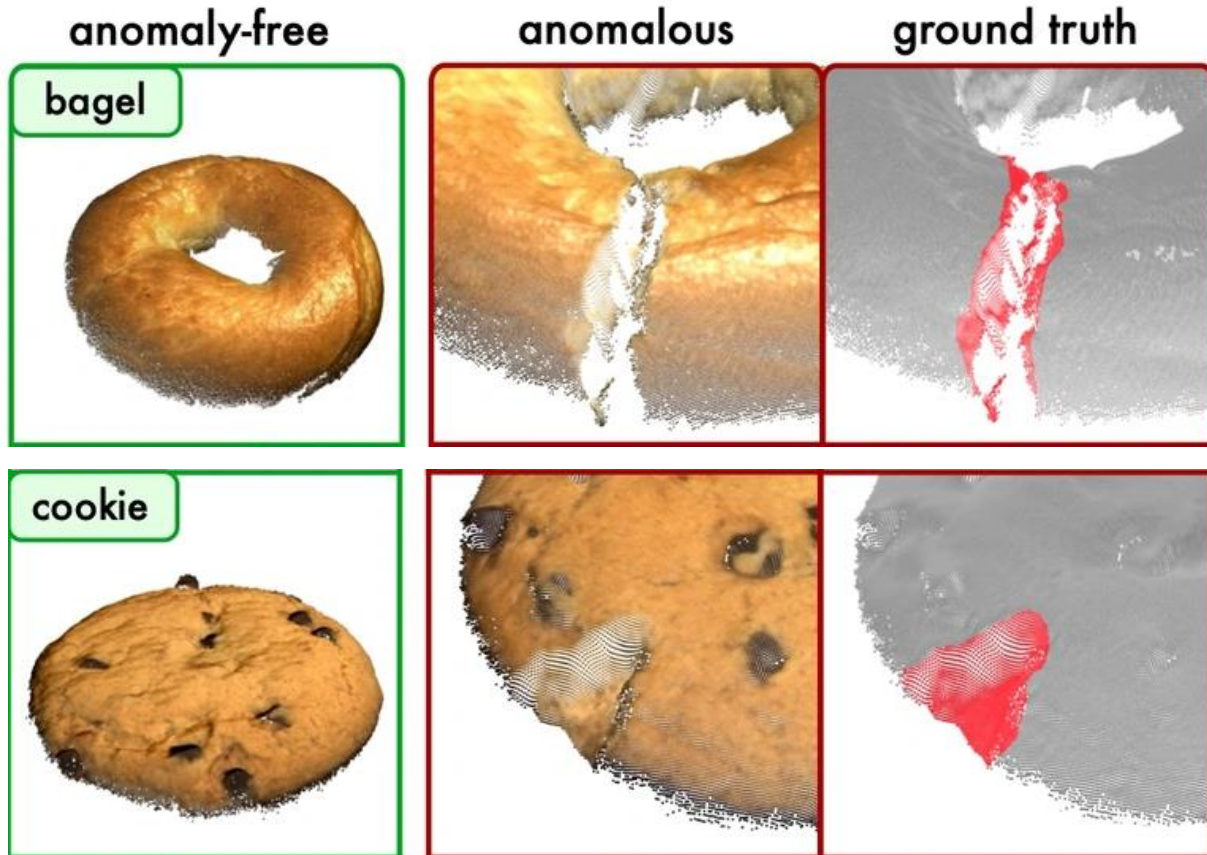
In some cases, anomalies are better identified as samples that do not conform to *low-dimensional manifolds*.

Such as *periodic texture* in 2D images





# Structure-based Anomaly Detection



["The MVTec 3D-AD Dataset for Unsupervised 3D Anomaly Detection and Localization"](#).

*Normal* data are locally approximated by parabolic shapes or even planes. *Anomalies* (cracks, spikes, holes) depart from these assumptions.

## Structure-based Anomaly Detection

The only assumption: the *analytical expression* of the model family  $\mathcal{F}$  describing normal data.

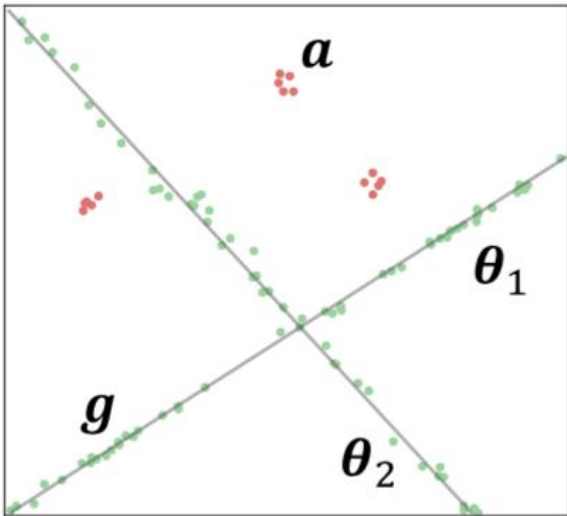
- Assume a *local structure* e.g., “locally approximated by planes”.

Truly “**unsupervised settings**” no training data needed, neither pre-trained models!

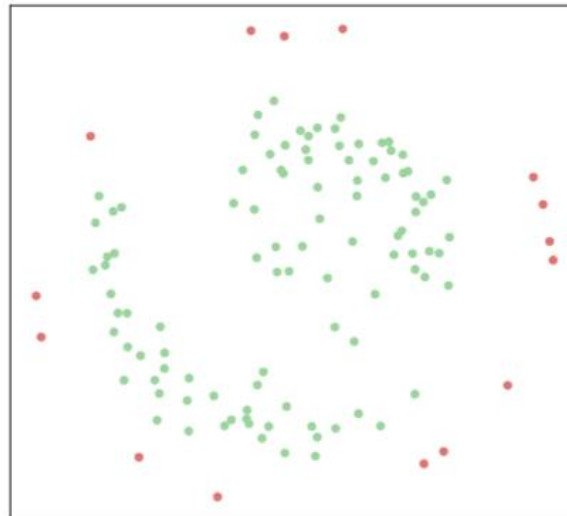
# PIF: Preference Isolation Forest

PIF has two major components:

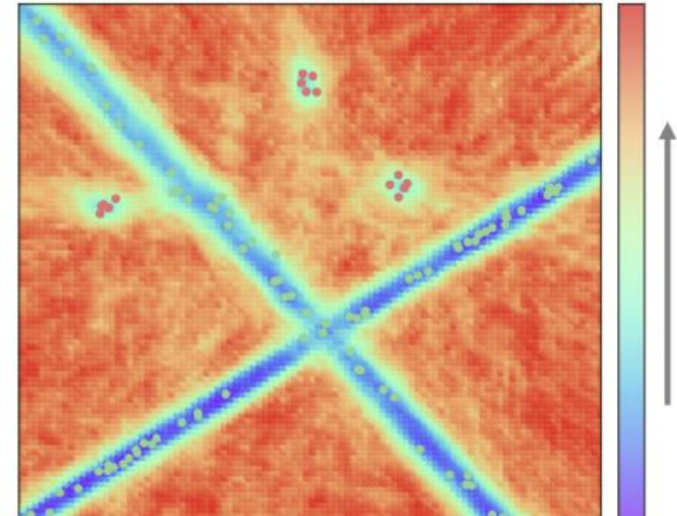
- i) **Preference Embedding**, that maps input data to the (high-dimensional) Preference Space, by fitting models from the family  $\mathcal{F}$ .
- ii) **Preference Isolation**, to detect anomalies as isolated points in the Preference Space.



(a) Input data  $X = G \cup A$ .



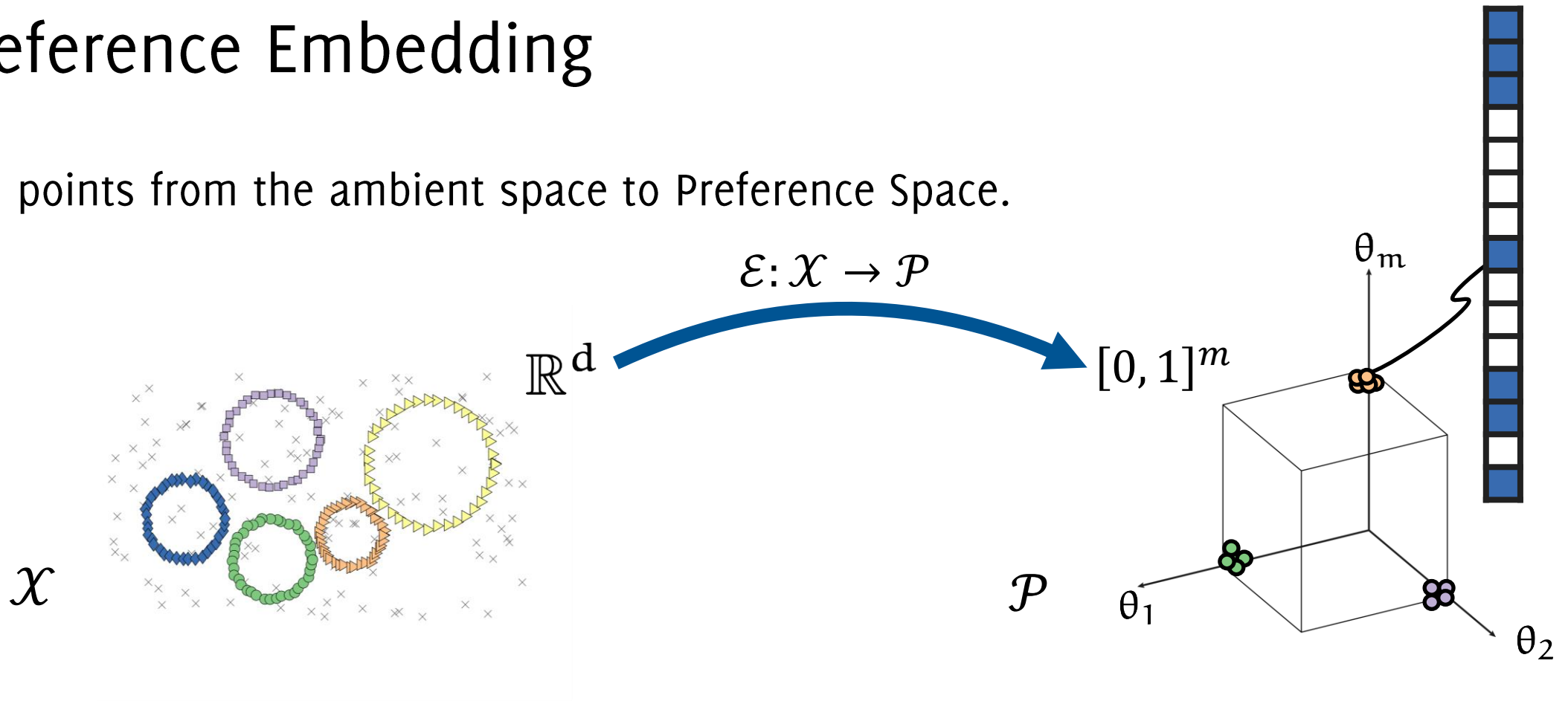
(b) Preference space  $\mathcal{P}$ .  
MDS visualizazzion



(c) Anomaly scores  $\alpha$ .

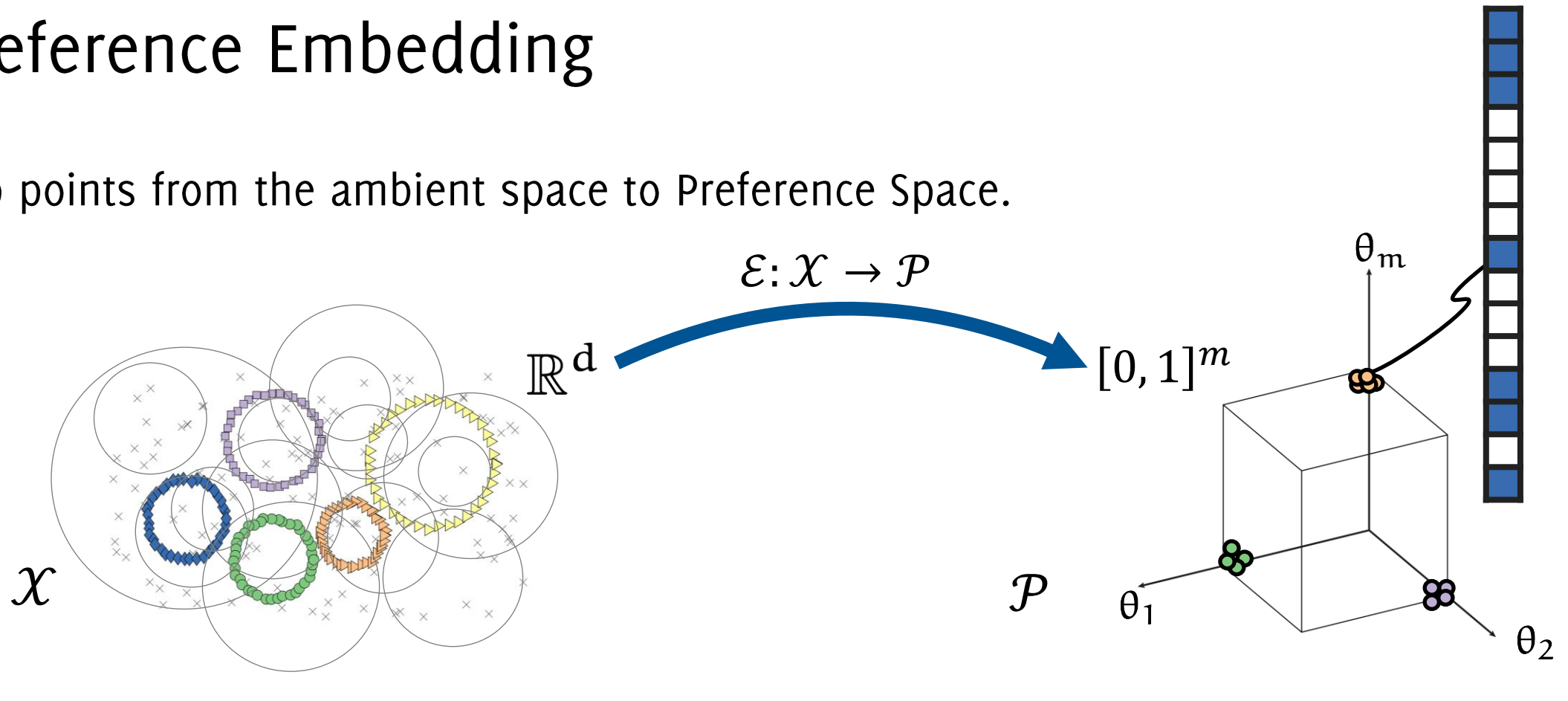
# i) Preference Embedding

Map points from the ambient space to Preference Space.



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Map points from the ambient space to Preference Space.



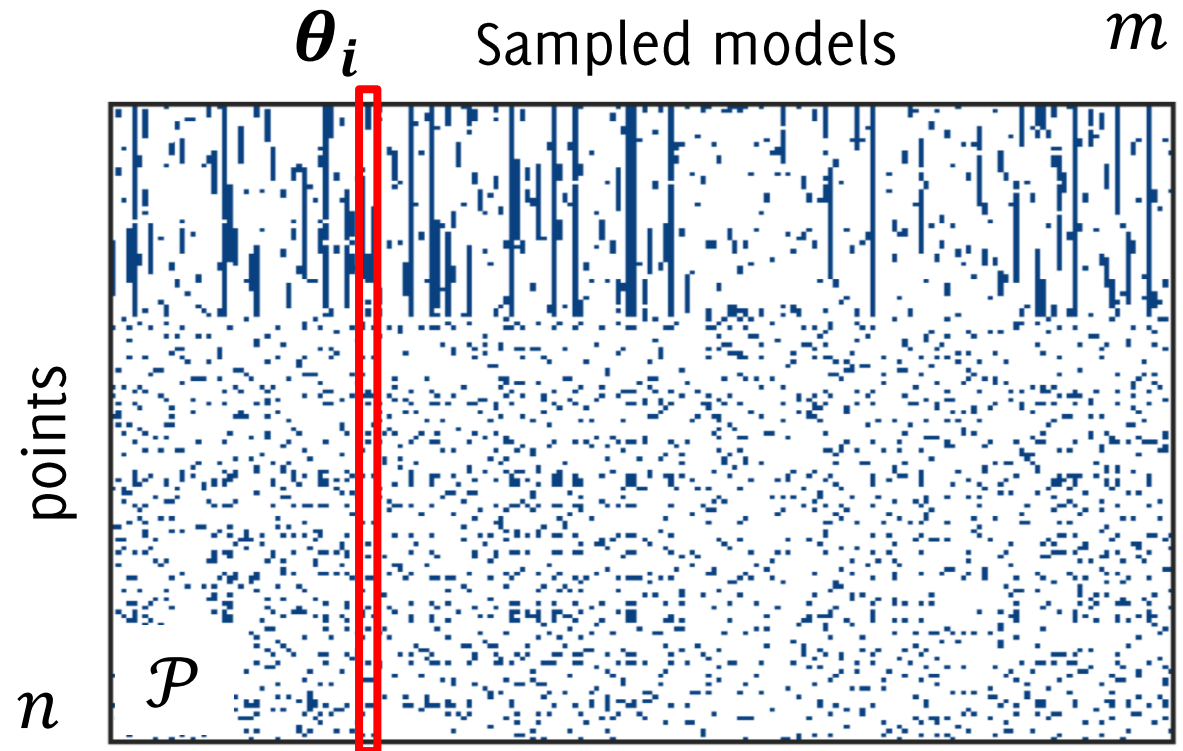
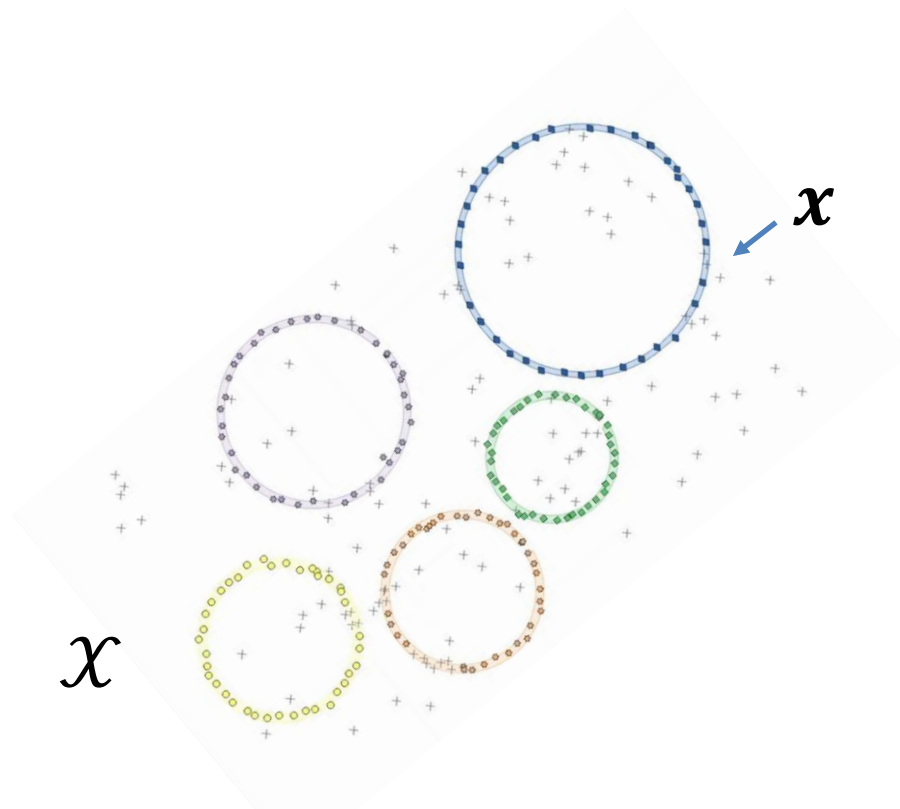
A set  $\{\theta_1, \dots, \theta_m\}$  of models from family  $\mathcal{F}$  are fit on  $\mathcal{X}$  by randomly sampling minimum sets (with  $m \gg 0$ ).



## i) Preference Embedding

To each model-point pair we associate a preference value in  $[0,1]$ :

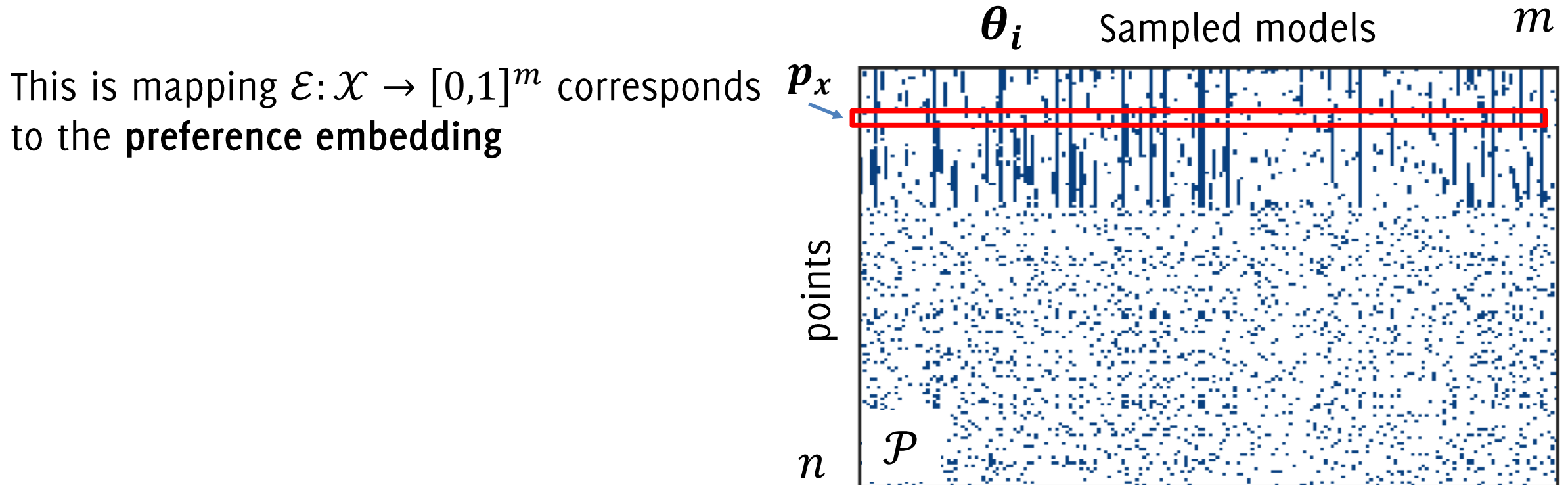
- 0 means  $\mathbf{x}$  does not belong to the model  $\boldsymbol{\theta}_i$ , namely  $|\mathcal{F}(\mathbf{x}, \boldsymbol{\theta}_i)| > \epsilon$
- $\approx 1$  means  $\mathbf{x}$  has a strong preference for  $\boldsymbol{\theta}_i$ , namely  $|\mathcal{F}(\mathbf{x}, \boldsymbol{\theta}_i)| \approx 0$



# i) Preference Embedding

The preference embedding consists in associating each point  $\mathbf{x}$  to all its preferences w.r.t. the  $m$  sampled models.

$$\mathcal{E}: \mathbf{x} \mapsto \mathbf{p}_x \quad \text{such that } [\mathbf{p}_x]_i = \begin{cases} \phi(\mathcal{F}(\mathbf{x}, \boldsymbol{\theta}_i)), & \text{if } |\mathcal{F}(\mathbf{x}, \boldsymbol{\theta}_i)| < \epsilon \\ 0, & \text{otherwise} \end{cases}$$

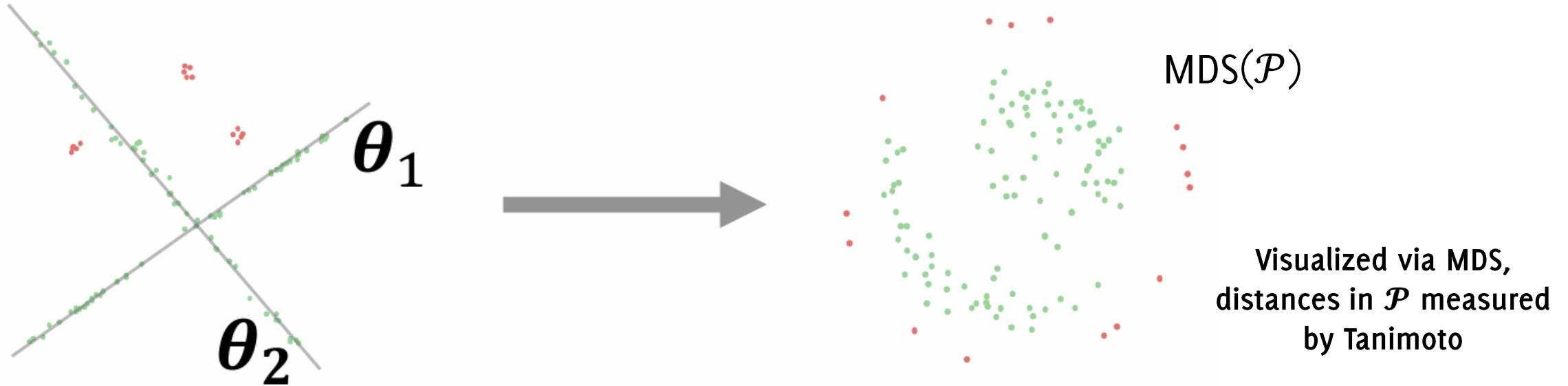


## ii) Preference Isolation

In the preference space:

- genuine points cluster together
- anomalies appear far apart

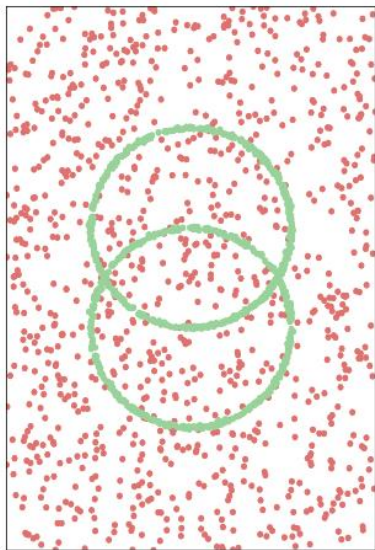
**Idea: use isolation-based AD schemes to detect anomalies**



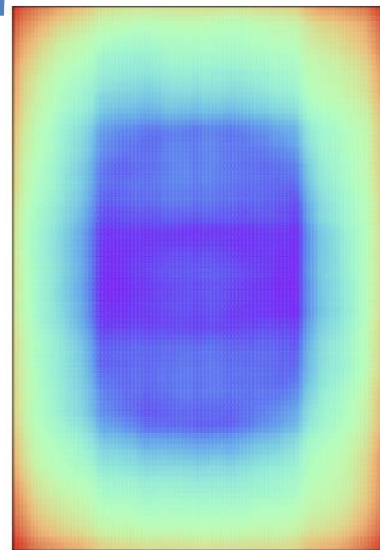
# Preference Isolation

Any **isolation-based AD** can be used in  $\mathcal{P}$ , returning an anomaly score.

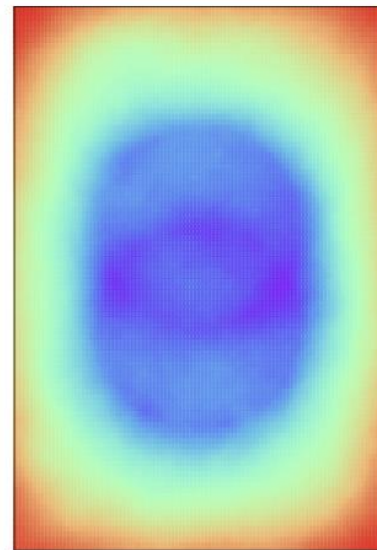
Anomaly Scores computed in the Preference Space



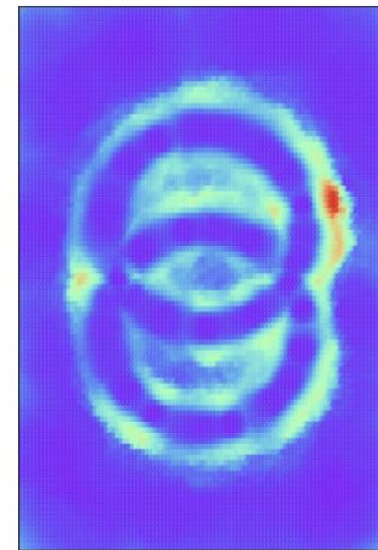
(a) Input data.



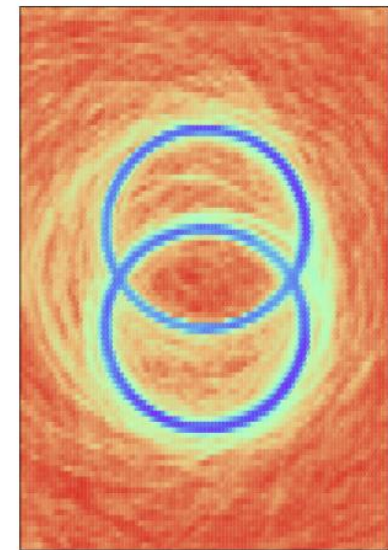
(b) iFOR [14]



(c) EiFOR [15]



(d) LOF [13]



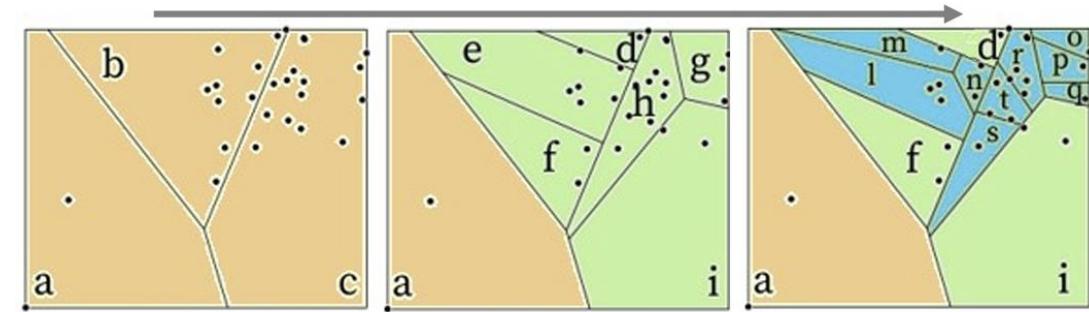
(e) PIF

However, distances in the preference space are not the same as in  $\mathcal{X}$ , and isolation-based methods based on Euclidean distance fall short in AD.

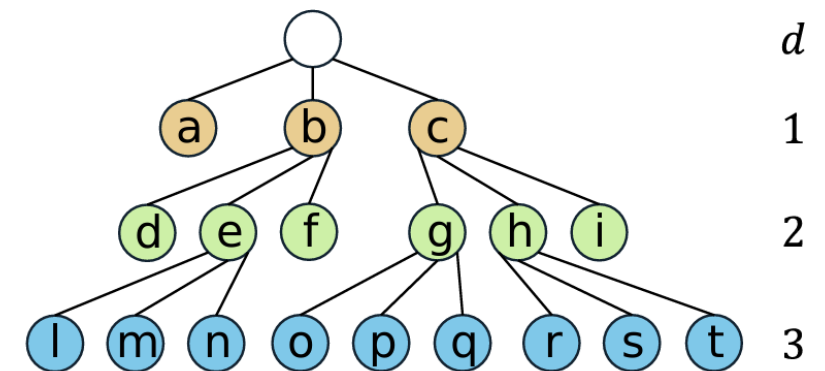
# Preference Isolation

Distances in the Preference Space are better measured by *Jaccard*, *Tanimoto*, *Ruzicka* distances

To effectively detect anomalies in  $\mathcal{P}$  we construct a forest of Voronoi tessellations (Voronoi i-Tree), based on any of the above distances



An efficient variant, where trees are constructed based on hashing of the Ruzicka distance has been also investigated.





# Anomaly Score

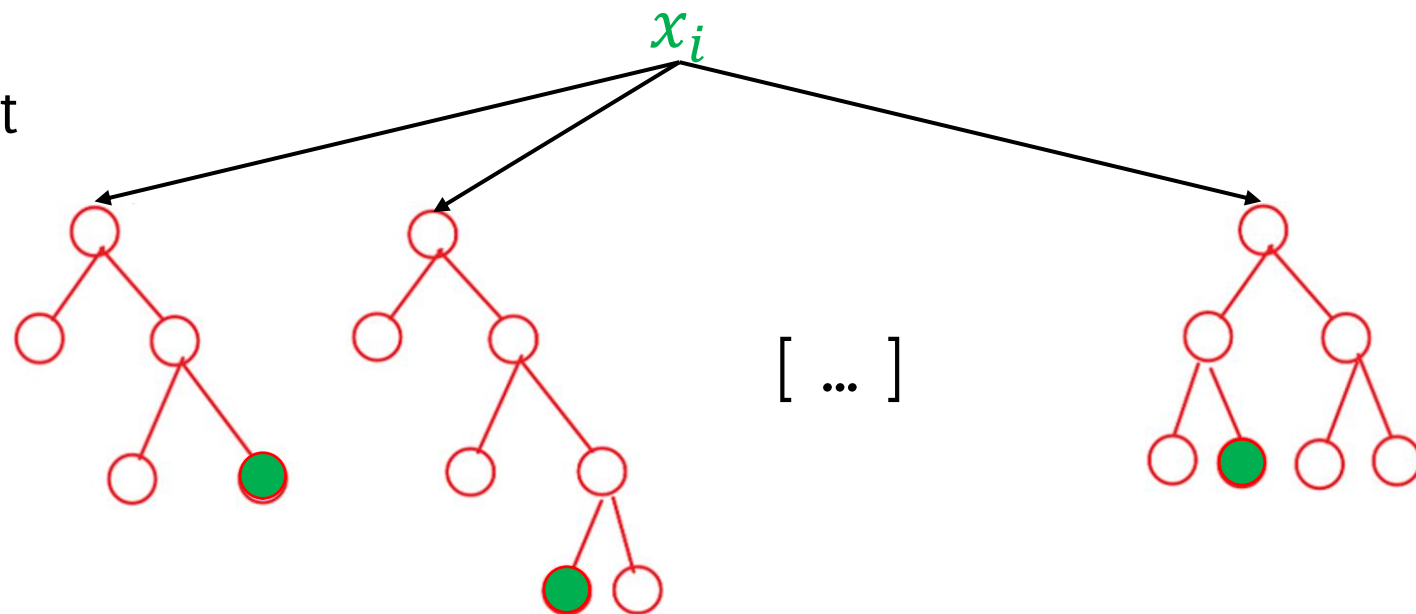
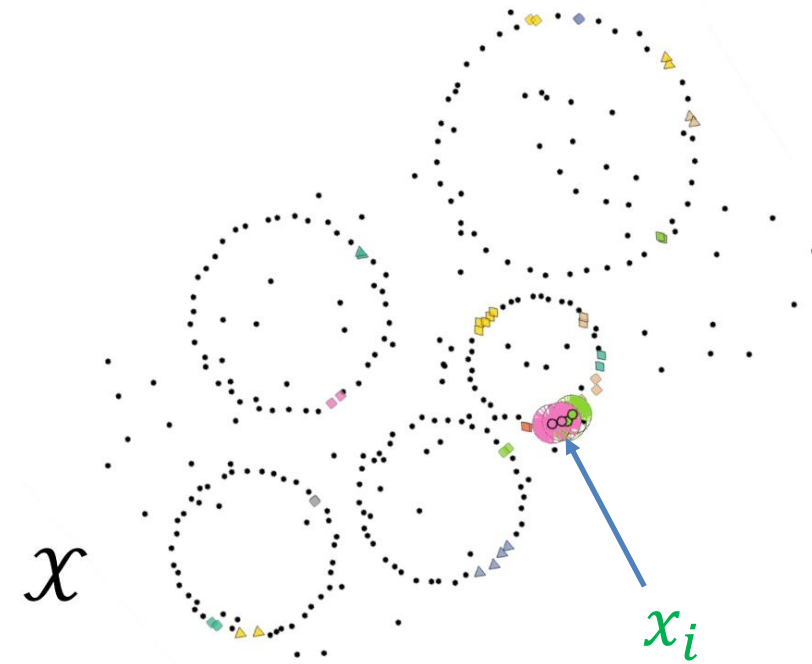
The anomaly score is computed from the forest  $\psi$  as in IFOR,

$$\alpha_{\psi}(\mathbf{x}) = 2^{-\frac{E[D(\mathbf{x})]}{c(\psi)}}$$

where

- $E[D(\mathbf{x})]$  is the average depth reached by a point
- $c(\psi)$  is a normalizing factor depending on the forest  $\psi$

This score underpins the intuition that normal samples reaches leaves at larger depths than anomalous ones, which can be easily isolated by a few splits



# Anomaly Score

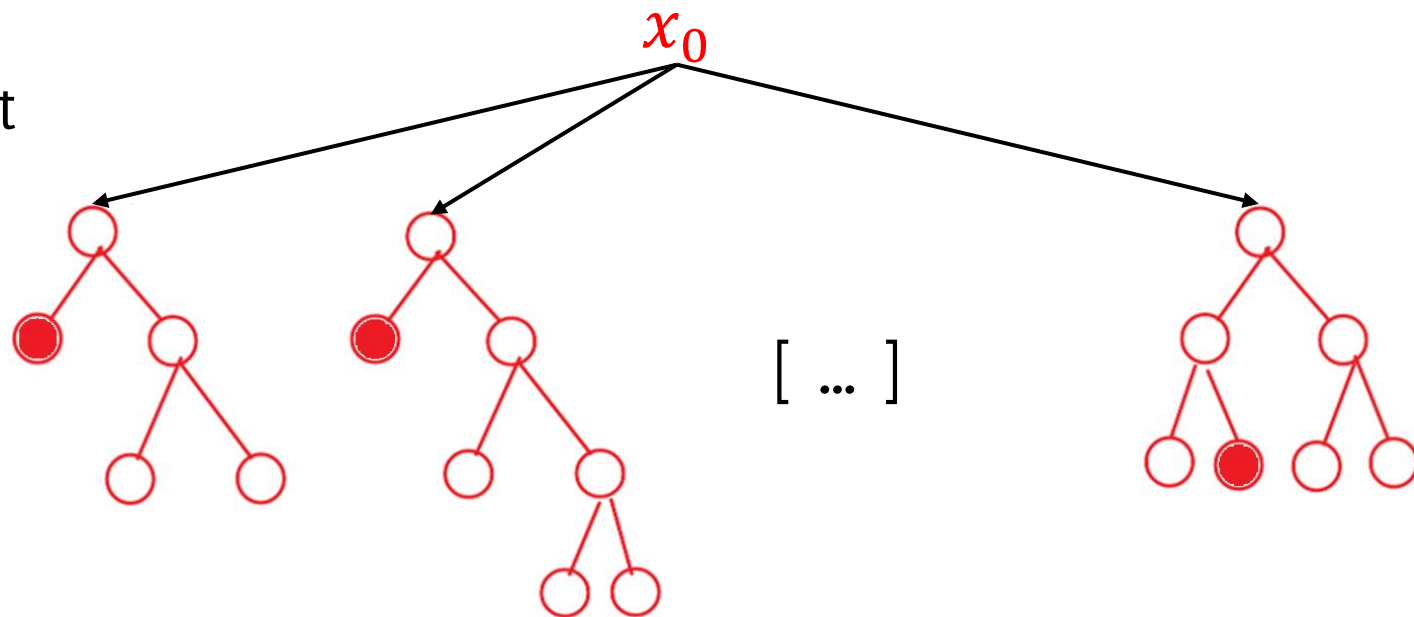
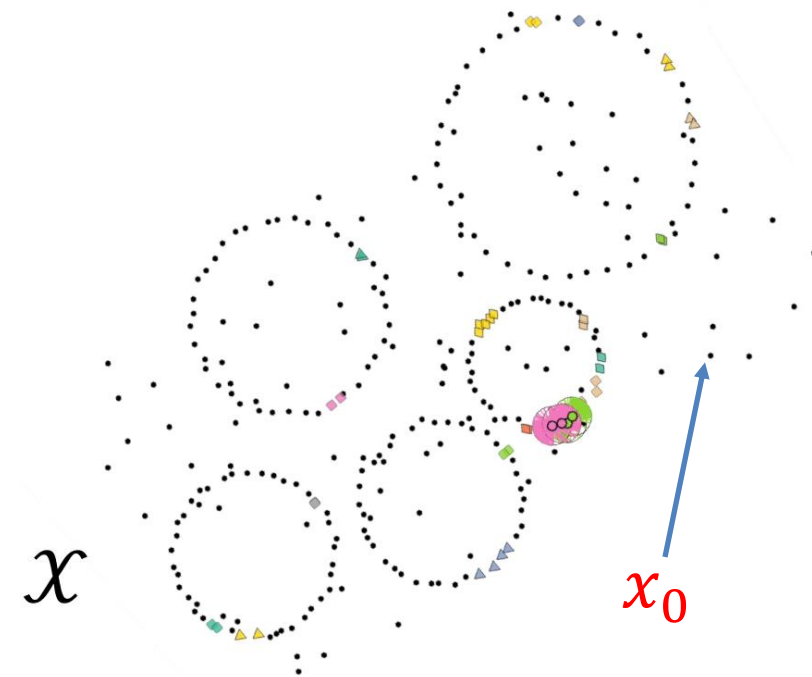
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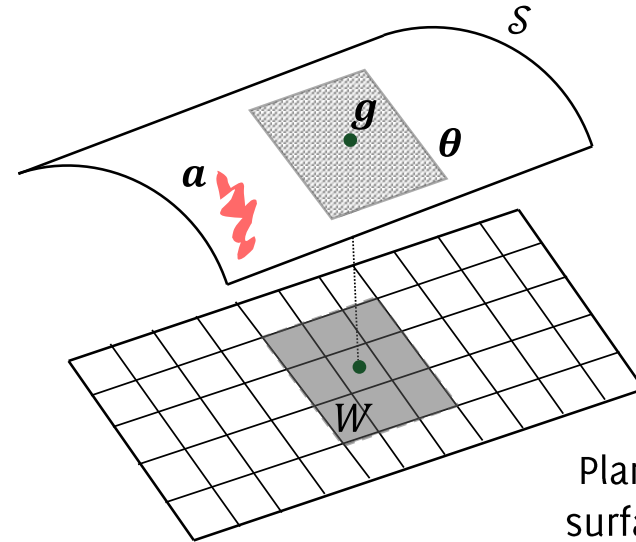
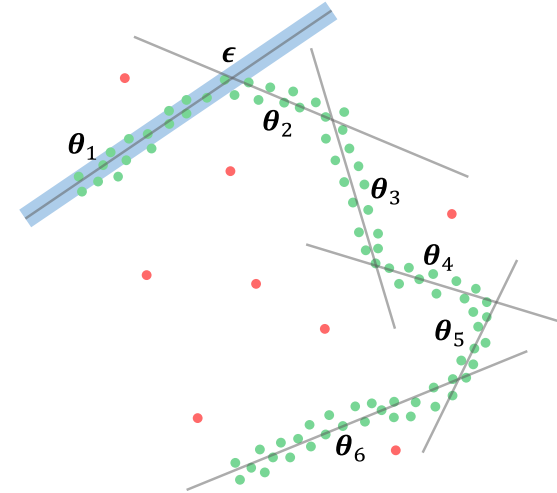
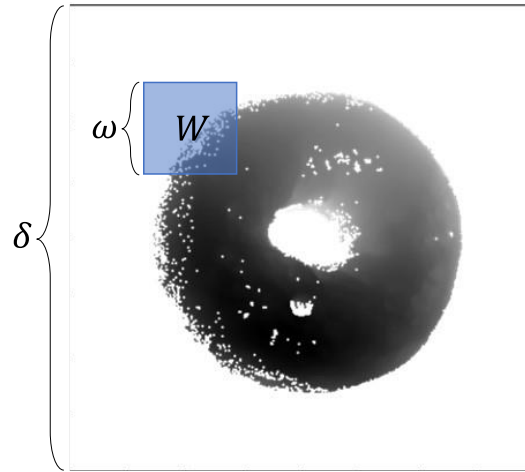
This score underpins the intuition that normal samples reaches leaves at larger depths than anomalous ones, which can be easily isolated by a few splits



# Sliding PIF

Sometimes only *local* knowledge about normal model family  $\mathcal{F}$  is available.

We can sample models  $\theta_i$  belonging to a *local* family by performing PIF in a sliding-window fashion.

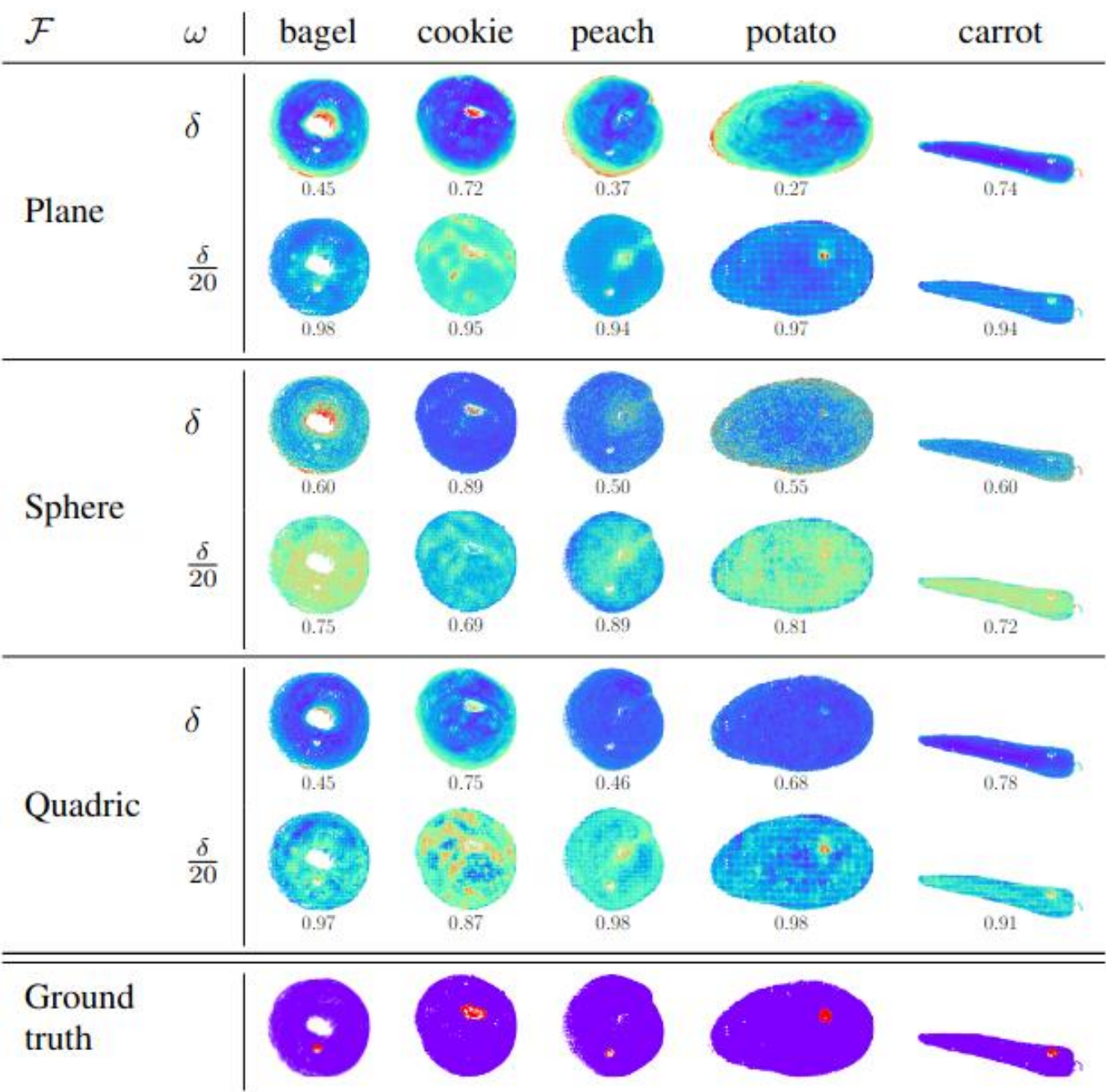


Planes to approximate a surface  $\mathcal{S}$  belonging to an unknown family  $\mathcal{F}$ .

# Sliding PIF

It is sufficient to opt for any model family  $\mathcal{F}$  matching the normal manifold's *dimensionality* to estimate  $\{\theta_1, \dots, \theta_m\}$  for the Preference Embedding.

We might generalize to broader families of matching dimensionality and allow for *more flexible* local approximations.



# Concluding Remarks

Anomaly detection is a very hot research problem with multiple applications

The field underwent the «deep learning tsunami», but still faces a dual nature

- Model based
- Statistical

That need to be taken into account when designing novel solutions.

